JLAB-TN-02-002
10 January 2002

A Magnet Field Quality Limitation On ERL Performance

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Introduction

Imperfections in magnet field quality have long been recognized as a potential source of performance limitations in many types of accelerator. In transport lines and linacs, field inhomogeneities can lead to beam envelope mismatch, orbit-dependant optics, phase space distortion, and emittance degradation. The mechanism for such effects is simple – field deviations lead to positional-dependent bending of portions of the beam in a manner differing from design and/or that experienced by other portions of the beam. The resulting unanticipated spread in beam angular distribution manifests itself as a focusing error (if linear) or a phase space distortion (if nonlinear). See Figure 1.

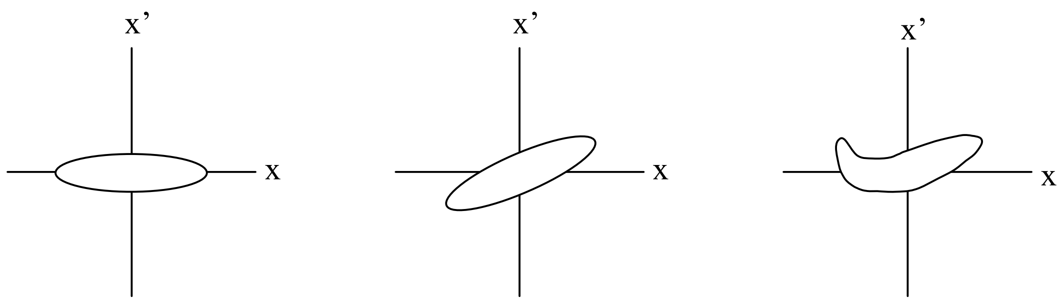

Figure 1: Ideal (left), linearly perturbed (center) and nonlinearly perturbed (right) beam phase space.

In conventional linacs and transport lines, these effects are often of significance in only the transverse portion of phase space. In recirculating and energy recovering linacs they can however be longitudinally deleterious as well, by virtue of the six-dimensional nature of the beam dynamics (what would be called "synchro-betatron coupling" in the world of storage rings). As these systems relay upon various compaction management schemes to provide longitudinal phase space control, the presence of unexpected sources of angular error can lead to unanticipated changes in bunch length and RF phasing. This in turn can alter the energy spread, thereby limiting performance – either at full energy (for example, by generating excessive energy spread at user experiments) or after energy recovery (by yielding an unmanageably large energy spread at the end of the machine).

In this note, we discuss the connection between field-error induced transverse angular spread and longitudinal behavior, and estimate its impact on the performance of an ERL by evaluating the energy spread after energy recovery when uncompensated field inhomogeneities are present.

Generation of Field Inhomogeneity-Driven Energy Spread

Consider the application over a length l of an error field ΔB to a portion (hereafter, referred to as a "filament") of a beam. This will result in unintended bending of the filament through an angle $\delta\theta = \Delta Bl/B\rho$. Should this angle be generated at a location from which the transfer matrix to reinjection has a nonzero M_{52} (as it generally will be, if the error is in a bend or other recirculation arc magnet), a path length error $\delta l = M_{52}\ \delta\theta$ will evolve. When reinjected, the beam filament will thus no longer be synchronous with the nominal RF phase; instead, it will experience a phase offset $\delta\phi = 2\pi\delta l/\lambda_{RF}$.

The energy recovered by deceleration at phase ϕ through a linac with energy gain E_{linac} is $E = E_{linac}\cos\phi$; the energy shift resulting from a phase offset $\delta\phi$ from a nominal phase set point ϕ_0 is therefore as follows.

$$\Delta E = E_{linac}\left[\cos(\phi_0 + \delta\phi) - \cos(\phi_0)\right] \cong -E_{linac}\sin\phi_0\ \delta\phi$$

It is assumed that energy compression is desired during energy recovery, and that deceleration (and, for that matter, acceleration) therefore occurs off-trough (crest). If the acceleration/energy recovery is on crest/trough ($\phi_0 = 0$), the sine goes away and the energy offset is quadratic in $\delta\phi$. Folding all the preceding expressions together yields an expression relating the imposed field error to the energy offset generated after energy recovery.

$$\Delta E = -\left(\frac{2\pi M_{52}}{\lambda_{\mathrm{RF}}}\right) E_{linac}\sin\phi_0 \left(\frac{\Delta Bl}{B\rho}\right) \qquad (1)$$

The filament thus ends up at the "wrong" final energy, offset by an amount depending on the RF wavelength, the field quality (ΔBl), the details of the transport (M_{52}), and the linac energy gain (E_{linac}). Given that there will be variations in field error across the full beam, different filaments will be transported to different final energy offsets – thereby increasing the energy spread of the beam after energy recovery. By viewing ΔB as a bounding value or tolerance on the field variation and M_{52} as an average value for the compactional term, we can then interpret ΔE as an estimate of the final energy spread.

Equation (1) can be rewritten to emphasize the effect of each of the various contributions. M_{52} is, for example, constrained by the symplectic condition to be a combination of dispersive and betatron components of the matrix:

$$M_{52} = M_{22}M_{16} - M_{12}M_{26}.$$

Control of final momentum spread is therefore promoted by the use of small dispersion and betatron function values. Use of a lower linac energy gain also reduces sensitivity. The impact of dipole field quality is made clearer by noting $(\Delta Bl/B\rho) = (\Delta B/B)(l/\rho) = (\Delta B/B)\ \theta$, where θ is the bend angle. The induced energy offset is then

$$\Delta E = -\,[(2\pi M_{52}/\lambda_{RF})E_{linac}\sin\phi_0\,](\Delta B/B)\theta\,, \qquad (1')$$

which is directly proportional to the relative field error $\Delta B/B$. The total angle will typically be π (such as in the energy recovery transport of an FEL driver) or 2π (as in a generic ERL recirculator); to maintain a fixed energy spread after energy recovery, the relative field error must therefore *decrease* as the linac energy *increases*.

This latter observation implies yet another viewpoint. Note that the energy spread at the dump depends on $E_{linac}/B\rho$. For injection energies small relative to the linac energy the rigidity will however be approximately 33.3564 kg-m/GeV/c × E_{linac}. The energy error after energy recovery is thus *independent* of the full energy, and is influenced by only the lattice parameters, RF wavelength (M_{52}/λ_{RF}) and the absolute error integral ΔBl.

$$\Delta E = -\left(\frac{2\pi M_{52}}{\lambda_{\mathrm{RF}}}\right)\sin\phi_0\left(\frac{\Delta Bl}{33.3564\ \mathrm{kg\text{-}m/GeV}}\right) \qquad (2)$$

The error field integral producing a specific energy error (energy spread) at the dump is thus independent of the linac energy. As the machine full energy increases (implying increased total field integral required to transport the higher energy beam), the tolerable *relative* error integral will decrease, as noted above.

Discussion

The coupling of transverse steering errors into the longitudinal motion can potentially seriously constrain the performance of recirculating linacs and ERLs. This is not limited to the generation of unanticipated energy spread after energy recovery that is discussed above. Field errors in recirculation transport during acceleration can lead to bunch lengthening and result in growth of momentum spread in any recirculating linac. Synchrotron radiation excitation, with an associated shift in energy at dispersed or compactional locations of the beam transport, will drive the evolution of bunch length errors (longitudinal emittance) in a fashion similar to the degradation of transverse emittance, again leading to growth of momentum spread at full, as well as recovered, energy.

These effects all encourage the use of "better" magnets, lower linac energy gains (between transport system modules with compaction management [1]), and smaller M_{52}s (dispersions, beam envelopes). We note that the effect of poor magnet field quality is not "undoable" – the steering induced by field errors can be corrected or compensated; it does however require provision for this compensation. Diagnostics (for example, phase transfer function measurement systems) and correction knobs (multipole correctors) should be made available if it is not possible to achieve the desired performance with the magnet field quality available within the constraints imposed by the system budget.

The preceding treatment only mentioned in passing the case of on crest acceleration and energy recovery. In such cases, the resultant energy spread is linear in the linac energy gain and quadratic in the phase (or field integral) error, suggesting that the product (which goes as linac energy divided by the square of the beam rigidity) will *decrease* with increasing linac energy. This would imply that on crest/in trough acceleration/energy recovery is quite desirable. This is in fact often the cast in recirculating, non-energy recovering linacs such as CEBAF, which accelerate a short, small momentum spread bunch on crest so as to take maximum advantage of the available gradients and to limit growth in bunch length and momentum spread. However, for higher charge-state machines (such as FEL drivers and light sources) it may be preferable to accelerate and energy recover off crest or out of trough. Off crest acceleration allows transport of longer bunches without undue excitation of wakefields and CSR; the bunch is compressed only where the user requires the time structure. Extraction of large amounts of power from the beam (either in an FEL, as a CSR source, or as a source of incoherent synchrotron radiation) will typically lead to generation of energy spread, which, when decelerated, will adiabatically antidamp to large relative energy spreads after energy recovery. SUPERCEBAF [2], for example, at 10+ GeV will likely generate relative momentum spreads in excess of 10^{-4} at full energy – corresponding to ~1 MeV rms or as much as 6 MeV full energy spread. This would be unmanageably large, were it energy recovered to ~10 MeV without energy compression.

The key point of this discussion is to note that between the limit of on-crest operation with no energy compression (with an associated horrible final energy spread due to adiabatic antidamping) and off-crest operation with energy compression (with an associated horrible final energy spread due to field-error induced bunch lengthening) there is, for any machine, likely to be an optimum at which the two sources of energy spread cross over and minimize the total final spread.

The JLab IR Demo and IR Upgrade Drivers parameter sets provide some insight into the magnitude of the effects under consideration. In both cases, λ_{RF} = 0.2 m and, on average, M_{52} ~ 1 m [3]. Energy recovery is performed at a phase offset of ~10^o so as to provide compression of the large energy spread generated during lasing. A final relative energy spread of order 1% is desired at the 10 MeV energy recovery dump, indicating the absolute energy spread is ~0.1 MeV. Equation (2) then indicates the field error integral must be of order

$$\Delta Bl \sim 0.1 \text{ MeV} \times 33.3564 \text{ kg-m/GeV} \times (0.2 \text{ m}/(2\pi \times 1 \text{ m}))/\sin 10^o = 61 \text{ g-cm} .$$

The induced energy spread for arbitrary field integral error is

$$\Delta E \sim ((2\pi \times 1 \text{ m})/0.2 \text{ m}) \sin 10^o /33.3564 \text{ kg-m/GeV } \Delta Bl = 1.64 \text{ keV/g-cm} \times \Delta Bl$$

The IR Demo recirculator bent the energy recovered beam through a total angle of ~$2\times180^o+8\times30^o=600^o$ at ~50 MeV. This represents a net field integral of 1.75×10^6 g-cm; the associated relative field error tolerance is thus

$$\Delta B/B \sim 61 \text{ g-cm}/1.75\times10^6 \text{ g-cm}=3.5 \times 10^{-5},$$

within a factor of 3 of the specified 10^{-4} [4]. The IR Upgrade recirculator will bend the energy recovered beam through ~$180^o+ 4\times43^o=352^o$ at up to 210 MeV, representing a net field integral of 4.3×10^6 g-cm and implying a field error tolerance of

$$\Delta B/B \sim 61 \text{ g-cm}/4.3 \times 10^6 \text{ g-cm} = 1.4 \times 10^{-5}$$

This is a factor of 6 from the specified 10^{-4}; it is well that we have included octupole order multipole corrections and that a dump beamline design with large acceptance (6%, [5]) has been developed.

We note that a CEBAF-ER [6] test would be a useful exercise to determine how extensive the impact of such effects actually is in a large system.

Notes/References

[1] The intent of this statement is most easily understood by analogy with transverse focusing systems. Strong focusing/large phase advance can be imposed by widely spaced, very short focal length quadrupoles. This is often an "unhappy" approach leading to overfocusing, error sensitivity, and sleepless nights for both machine operators and institutional managers. It can also be provided by the use of more closely spaced, weaker focusing. This is often a more robust solution. The use of a single, large energy gain linac preceded by one bunch compression or rotation transport system corresponds to the unhappy transverse focusing system; the use of multiple, shorter, lower energy gain linacs separated by a number of compaction management transport modules corresponds to frequent, more gentle transverse focusing.

[2] D. Douglas, "A SUPERCEBAF" Scenario Providing an Incremental Upgrade Path from 12 to 24 GeV", JLAB-TN-98-008, 19 March 1998.

[3] See, for example, Figure 2 of D. Douglas, "The Effect of Field Inhomogeneities In or Near the IR Upgrade Driver Compaction Management Elements", JLAB-TN-01-053, 12 November 2001.

[4] D. Douglas, "Error Estimates for the IR FEL Transport System", CEBAF-TN-96-035, 15 July 1996.

[5] C. Tennant, "IR/UV FEL Upgrade Energy-Recovery Dump Transport Design", JLAB-TN-01-038, 10 August 2001.

[6] D. Douglas, "CEBAF-ER: An Energy Recovery and Current Doubling Operational Mode for the Continuous Electron Beam Accelerator Facility", JLAB-TN-01-045, 19 September 2001.