29 January 2003
JLAB-TN-03-004

An Overview of Emittance Measurements for CEBAF-ER

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Abstract

A review of the standard quadrupole-drift-monitor set-up for emittance measurements is given and applied to CEBAF-ER. The critical problem of obtaining high-resolution emittance measurements in the extraction region is addressed. A two-stage solution, invoking a quad scan at 2L21 merely to get a handle on the emittance and a scan at 2L22 to get higher-resolution, is presented. In the latter case, an optics configuration to produce a round spot size at the harp allows scanning both horizontal and vertical planes simultaneously. Other issues related to the precision of emittance measurements, such as ensuring that a quad scan passes through a minimum of a beta function and knowledge of transfer matrices are also discussed in detail.

Introduction

With the successful operation of the IR FEL driver, Jefferson lab has demonstrated that a relatively high current (5mA) and relatively low energy (50 MeV) beam can be energy-recovered. The mission of CEBAF-ER is to demonstrate the feasibility of energy recovering a low current (100μA), high energy beam (845 MeV). Successful completion of the experiment will be the first step in the realization of energy recovering a high current *and* high energy beam. Because of the importance of this experiment, careful consideration has been given to the beam parameters that need to be monitored and how to obtain them. This paper details the process whereby we will measure the emittance and energy spread of the beam. We propose to make these measurements in the injector line, Arc 1, Arc 2 and in the extraction region. Perhaps the most important, and trickiest to achieve, is the emittance measurement in the extraction region. With this measurement we will be able to quantify the extent to which we have preserved the beam quality while energy recovering a nearly 1 GeV beam.

Emittance Measurements

The beam line configuration utilized most often to make emittance measurements consists of a quadrupole followed by a drift of length L to an observation point (see Figure 1).

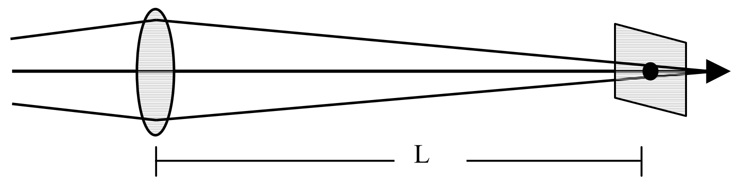


Figure 1: Schematic of quad-drift-monitor configuration to measure emittance

Denote the betatron functions just prior to entrance of the quad as β and α and those at the observation point by $\overline{\beta}$ and $\overline{\alpha}$. For the time being, we assume that we are in a low dispersion region so that we may write

$$\sigma = \sqrt{\overline{\beta}\varepsilon} \tag{1}$$

where σ is the rms beam size measured by the harp and ε is the rms geometrical emittance. We can relate the $\overline{\beta}$ to the beta function upstream, β, by knowing how the Twiss parameters propagate. One can show

$$\begin{pmatrix} \overline{\beta} \\ \overline{\alpha} \\ \overline{\gamma} \end{pmatrix} = \begin{pmatrix} M_{11}^2 & -2M_{12}M_{11} & M_{12}^2 \\ -M_{21}M_{11} & M_{12}M_{21} + M_{22}M_{11} & -M_{22}M_{12} \\ M_{21}^2 & -2M_{22}M_{21} & M_{22}^2 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} \tag{2}$$

where the M_{ij} are the elements of the transfer matrix that propagates beam from the quad to the harp. Multiplying the transfer matrices for a (thin lens) quad and a drift yields

$$\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix} = \begin{pmatrix} 1+kL & L \\ k & 1 \end{pmatrix} \tag{3}$$

Using equations (1), (2), and (3) the relationship between the beam size and the beta function prior to the quad entrance can be determined and is given by

$$\sigma^2 = \beta'\varepsilon = (1+kL)^2(\beta\varepsilon) - 2L(1+kL)(\alpha\varepsilon) + L^2(\gamma\varepsilon)\,. \tag{4}$$

Inspection of equation (4) reveals that the beam size squared varies quadratically with the quad strength, *k*. At present we have an equation with 3 unknowns; ($\beta\varepsilon$), ($\alpha\varepsilon$) and ($\gamma\varepsilon$). By changing the quad strength two times and re-measuring σ^2, we obtain a system of three equations and three unknowns from which the emittance can be extracted (see Appendix A). Note that only the matrix elements change in equation (4) since β, α, and γ are *upstream* of the quad. This process is known as "scanning the quad." In practice, the quad is scanned over many values of the strength. By fitting the data with a least-squares quadratic fit, one can obtain the unknowns. Using the procedure above, the values of ($\beta\varepsilon$), ($\alpha\varepsilon$) and ($\gamma\varepsilon$) are obtained. Use of the relation

$$(\beta\varepsilon)(\gamma\varepsilon) - (\alpha\varepsilon)^2 = \varepsilon^2$$

allows one to ultimately find the emittance. (The above relation is obtained by considering the determinant of a general transfer matrix written with the Courant-Snyder parameterization).

Several comments are in order. First off, we've assumed the simple configuration of a quad-drift-monitor. In practice this is the most effective way to make emittance measurements. But as can be seen from the previous analysis, the key to a good measurement is an accurate understanding of the transfer matrix between the place where the quad (or optics is changed) and the observation point. Uncertainty in the transfer matrix then introduces errors in addition to those associated with the measurements. More of this topic as applied to CEBAF-ER will be discussed later. The other point to be made is that we have assumed that the measurement was done in a low dispersion region so we could neglect the effect of energy spread on beam size. In general the beam size is written as

$$\sigma^2 = \beta\varepsilon + (\eta\delta)^2$$

where η is the dispersion and δ is the energy spread (= $\Delta E/E$) at the observation point. In the preceding discussion we've assumed $\beta\varepsilon >> (\eta\delta)^2$. It follows that if the observation point is in a high dispersion area, that we can assume the energy spread contribution to the beam size dominates $(\eta\delta)^2 >> \beta\varepsilon$, and write

$$\delta \approx \frac{\sigma}{\varepsilon} \tag{5}$$

We now have the information needed to make both emittance and energy spread measurements of the beam – both of which are useful for quantifying energy recovery in CEBAF.

Emittance Measurements for CEBAF-ER

During the CEBAF energy recovery experiment, emittance and energy spread will be measured in Arcs 1 and 2 and at the extraction region. The measurements in the arcs are, in principle, straightforward. In Arc 1 a harp will be placed in the spreader region (1E03) in an area of zero dispersion to allow emittance measurements. The arcs will be configured to the "medium dispersion" optics, which produces a maximum dispersion of 6 m [see Appendix B for CEBAF-ER optics]. The second harp in the Arc 1 will be placed at the maximum dispersion region (1A21) to allow energy spread measurements. In the previous section we noted that in an area where the beam size can be contributed primarily to the energy spread we could write down equation (5). This is only an approximation and to be correct we need to include the effects of the beta functions. That is,

$$\delta = \frac{1}{\eta}\sqrt{\sigma^2 - \beta\varepsilon}$$

where we can use the emittance from the previous measurement. Likewise, we can take the β obtained from the measurement at the first harp, propagate it downstream via equation (2) and use that value in the above expression. Caution must be taken however because there will be *two* 445 MeV beams in Arc 1 simultaneously (one from the deceleration through the north linac from 845 MeV and one from acceleration through the linac from 45 MeV). A scheme to separate and then "tag" the beams must be employed. One can steer the beam so as to differentiate the quad scans, but the problem of identifying which beam is the accelerating and decelerating beam remains. One way to solve this problem is to excite correctors at the end of Arc 2 [2]. The accelerated beam will not be affected and one can in essence, "wiggle" the decelerated beam for identification. This same configuration of harp placements is repeated in Arc 2; one harp in the spreader region (2E02) with zero dispersion and one in at the maximum dispersion region (2A21) [1]. Note that at any time there is only one beam traveling in Arc 2 since the energy-recovered beam has already been extracted.

The real challenge is obtaining these measurements on the energy-recovered beam in the extraction region. It was decided that a two-stage approach to this problem was necessary [3]. The first stage would be to scan the quad at 2L21 and using the harp just prior to the bending magnet of the dump chicane, obtain an emittance measurement (see Figure 2).

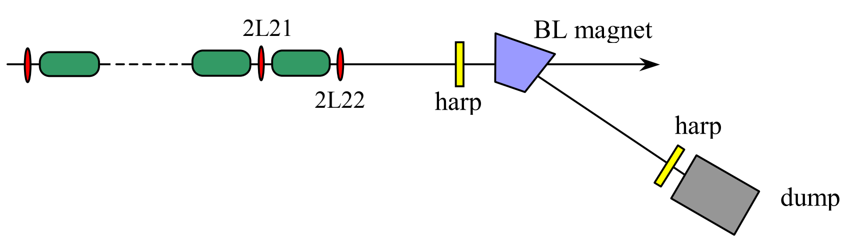


Figure 2: Diagram of extraction region at the end of the south linac

The harp located along the dump line is in a dispersive region and would be used for the energy-spread measurement. This procedure does not yield good resolution measurements for reasons that will be discussed shortly. Therefore, the second stage of this process, in order to yield a high-resolution measurement, requires scanning the quad at 2L22 with the optics changed substantially so as to produce a round spot size at the harp. The reasons for which will be discussed in subsequent sections.

Stage 1: Quad Scan at 2L21

As we noted previously, the beta functions at an observation point following a quad will vary as the quad strength squared. A typical quad scan will yield a data set like that shown in Figure 3a.

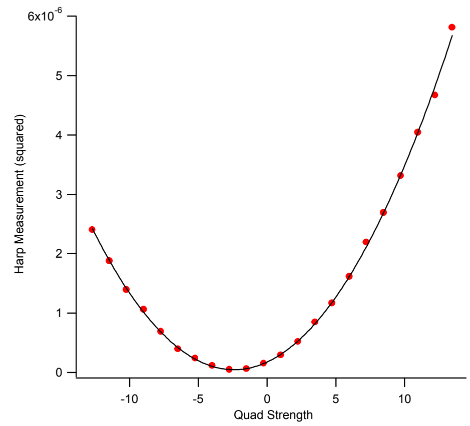


Figure 3: (a) a good emittance data set with a minimum of the beta function

As noted by others, and from the author's personal experience with previous emittance data, there are several "rules-of-thumb" one needs to consider when collecting emittance data [4]. Perhaps the most vital is the requirement that the beta function passes through its *minimum* during the scan. The problem is that a quad scan at 2L21 allows the horizontal beta function to pass through a minimum but not the vertical beta function (see Figure 4). Therefore we would only be able to measure the emittance in one plane with any confidence.

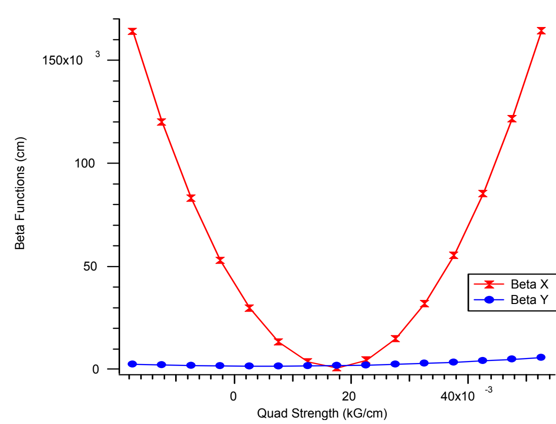


Figure 4: Quad scan at 2L21

We also noted that an acceptable measurement requires a good understanding of the transfer matrix from the point of where the optics is changed to the observation point. Therefore a quad scan at 2L21 implies we know the transfer matrix through a five-cell CEBAF superconducting accelerating cavity. Although we have several good models that describe the transfer matrix through such a cavity, it is not clear that we have the precision necessary to make a good emittance measurement. In addition to not knowing the transfer matrix to an acceptable degree of accuracy, the fact that the energy recovered beam at the end of the south linac has an energy of about 45 MeV makes it much more susceptible to cavity focusing. In addition care must be taken to avoid the cavity couplers inducing a coupling in the x-y motion. This means that even small beam displacements would translate to large measurement errors. Clearly it is more advantageous to use a quad-drift-harp set up rather than a quad-cavity-drift-harp set up, which is explained in the following section.

Consider figures 5a and 5b which display plots of the transfer matrix element M_{21} versus the phase for a single CEBAF cavity. Figure 5a represents the results for the first cavity of the last cryomodule in the south linac - which decelerates the beam on the second pass from 65 MeV to 62.5 MeV. Figure 5b represents the last cavity of the last cryomodule which decelerates the beam from 47.5 MeV to 45 MeV on the second pass. The analysis was performed using DIMAD, Optim's π-mode model and using Optim to calculate the transfer matrix using integration through a longitudinal field profile. Inspection of figure 5b shows good agreement between the various methods of calculating the effective focal length ($1/f = M_{21}$) of the cavity. Compared to the length of the cavity, the focal length is very large (~500 m for a phase of $\pm$180 degrees) so that cavity focusing should not be a major issue. We expect that at higher beam energies, the agreement between the methods of calculating the M_{21} should improve. Yet while DIMAD and Optim are shown to have better agreement in figure 5a, it is puzzling that the result from the field profile shows an appreciable discrepancy. These discrepancies will be discussed in a future Tech Note.

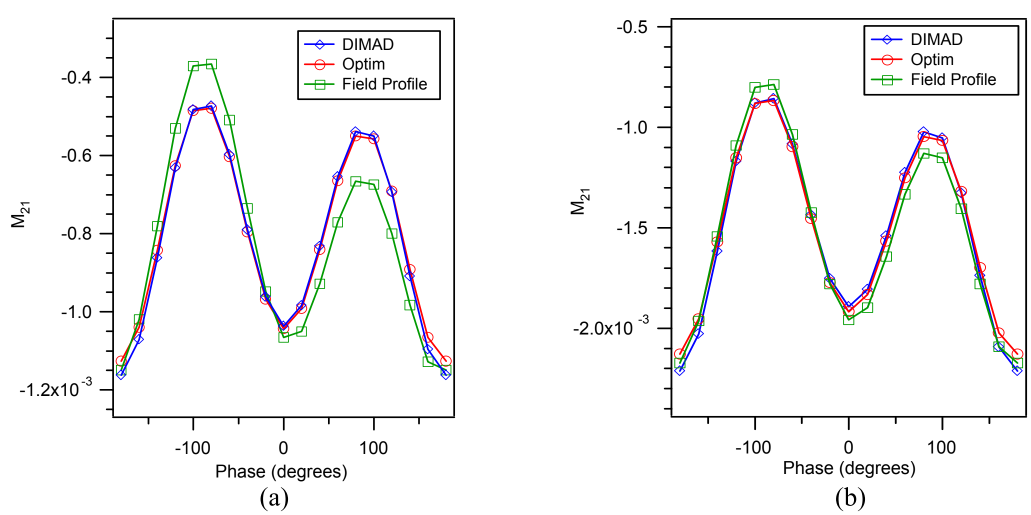


Figure 5: Transfer matrix element M21 as a function of phase for a single CEBAF cavity for deceleration from (a) 65 MeV to 62.5 MeV (b) 47.5 MeV to 45 MeV.

One final comment is in order. In the preceding analysis we have been comparing various *models*. In fact one should be comparing the performance of the *machine* with the models. This can be accomplished by taking differential orbit data in the injection line. By giving a kick to the beam and taking positional data, one can compare the resulting orbits to what each model predicts. Once the differential orbit has been taken, it is a simple matter to compare the machine performance with the model predictions in Optim.

Stage 2: Quad Scan at 2L22

The procedure described above as stage 1 serves only to give us a handle on the emittance measurements. Attempting to energy recover a nearly 1 GeV beam provides numerous operational challenges. Once sufficient control and understanding of the beam is achieved high-resolution emittance measurements can be made. By scanning the quad at 2L22 we immediately eliminate the problem with the transfer matrix of an accelerating cavity. Furthermore, it was noted that if one could produce a round spot size at the harp, then by scanning the quad we could measure the emittance in both planes simultaneously [5]. To produce a round spot size the optics must be modified to make the β_x equal to β_y and α_x equal to α_y at the harp. Doing so, however, creates a large spike in the β functions prior to the harp (see Figure 6).

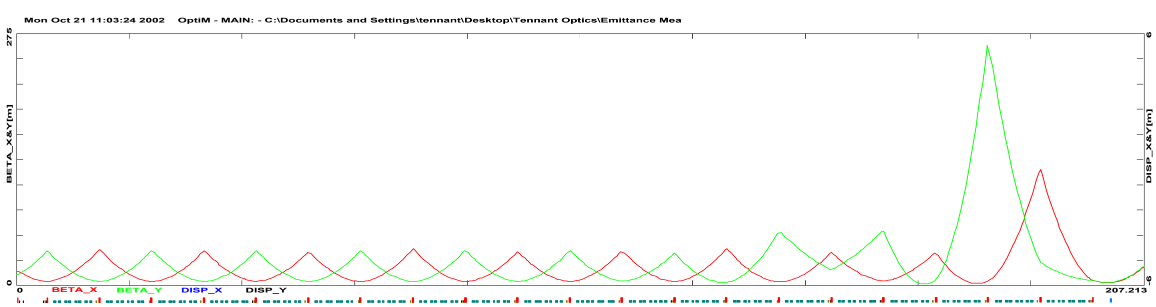


Figure 6: Decelerating beam through south linac with optics configured to yield a round spot size at the harp

These large spikes (and hence, large beam size) are precisely why we elected to go through stage 1 before attempting these higher-resolution measurements. A plot of the beta functions versus the quad strength of the quad at 2L22 is shown in Figure 7. Note that both beta functions pass through a minimum ensuring a good quadratic fit.

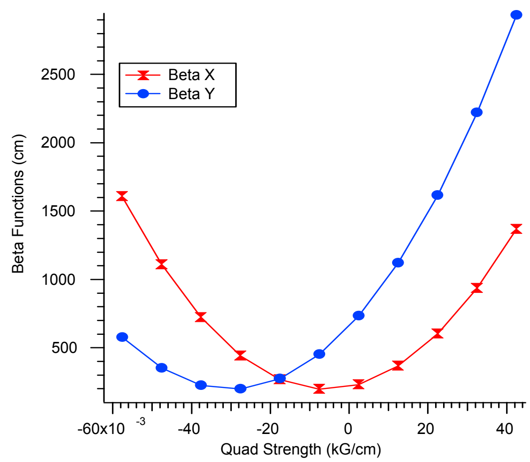


Figure 7: Quad scan at 2L22

Conclusions

A procedure for obtaining the critical emittance measurements in the extraction region has been presented. The most logical course of action seems to be to get an emittance measurement by scanning the quad at 2L21, although we can only measure the horizontal emittance and even then, we expect to have large errors associated with it. It was suggested that taking differential orbits in the injection line may help to reduce the error by indicating which model most accurately describes the transfer matrix through an accelerating cavity. Once we have a sufficient handle on the beam, we can alter the optics slightly so that a quad scan at 2L22 will yield beta functions such that a high resolution emittance measurement in both the horizontal and vertical plane can be made. (A solution addressing the problem of measuring the energy spread in the extraction region is given in JLAB-TN-03-005.)

Acknowledgements

This note is merely the collection and summarization of information collected from among the following people during the course of CEBAF-ER meetings: Kevin Beard, Alex Bogacz, Yu-Chiu Chao, David Douglas, Lia Merminga, and Byung Yunn.

Appendix A: Least-Squares Fit to Extract Emittance

Equation (4) can be written in terms of the σ-matrix elements as

$$\sigma^2 = \sigma_{11}(1+kL)^2 + \sigma_{21} 2L(1+kL) + \sigma_{22}L^2 \tag{1'}$$

where

$$\sigma^M \equiv \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \beta\varepsilon & -\alpha\varepsilon \\ -\alpha\varepsilon & \gamma\varepsilon \end{pmatrix}$$

We can rewrite equation (1') as a system of the form

$$y_j = Ax_j^2 + Bx_j + C$$

where we have defined

$$y_j = x^2_{measured,j}$$
$$x_j = (1 + k_j L)$$

To do a quadratic least-squares fit, we define the error function,

$$E \equiv \sum_{j=1}^{N} \left[y_j - \left(Ax_j^2 + Bx_j + C \right) \right]^2$$

and minimize it with respect to A, B and C. That is,

$$\frac{\partial E}{\partial C} = 2\sum_{j=1}^{N} \left[y_j - \left(Ax_j^2 + Bx_j + C \right) \right](-1) = 0$$

$$\frac{\partial E}{\partial B} = 2\sum_{j=1}^{N} \left[y_j - \left(Ax_j^2 + Bx_j + C \right) \right](-x_j) = 0$$

$$\frac{\partial E}{\partial A} = 2\sum_{j=1}^{N} \left[y_j - \left(Ax_j^2 + Bx_j + C \right) \right](-x_j^2) = 0$$

The above equations can be can be expressed in matrix notation (where the sum from *j=1* to *N* is implied)

$$\begin{pmatrix} N & \sum x_j & \sum x_j^2 \\ \sum x_j & \sum x_j^2 & \sum x_j^3 \\ \sum x_j^2 & \sum x_j^3 & \sum x_j^4 \end{pmatrix} \begin{pmatrix} C \\ B \\ A \end{pmatrix} = \begin{pmatrix} \sum y_j \\ \sum y_j x_j \\ \sum y_j x_j^2 \end{pmatrix} \tag{2'}$$

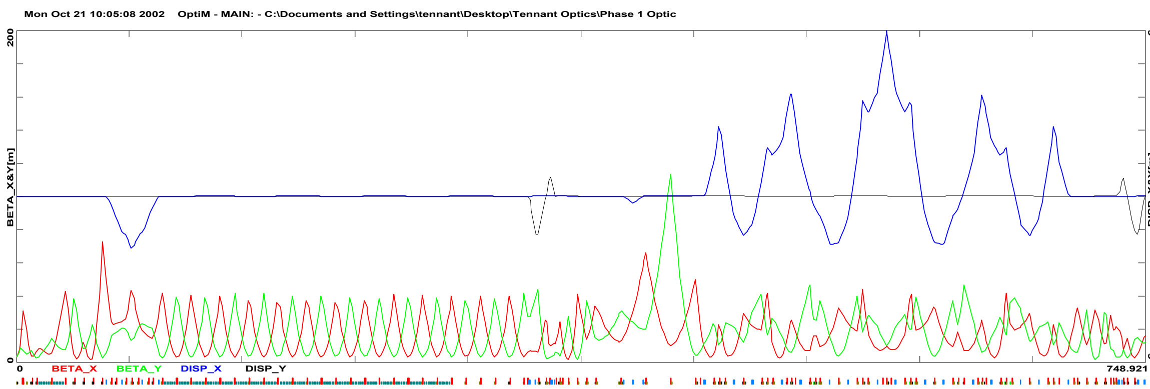
The matrix expression in equation (2') can be inverted to yield expressions for the coefficients (*A,B,C*) in terms of the measured quantities. And from equation (1')

$$\begin{aligned} \sigma_{11} &= A \\ \sigma_{21} &= \frac{B}{2L} \\ \sigma_{22} &= \frac{C}{L^2} \end{aligned} \tag{3'}$$

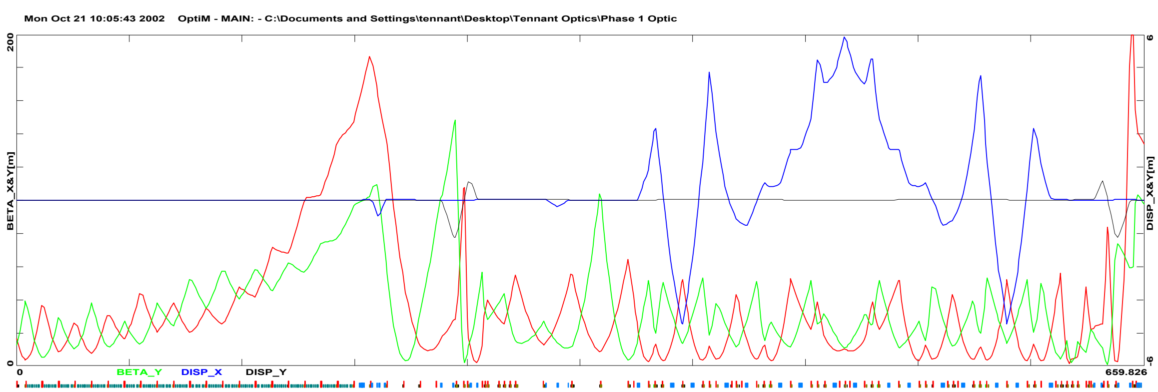
The emittance is defined as the square-root of the determinant of the σ-matrix, $\varepsilon = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{21}^2}$, which can be found in terms of the measured quantities using equations (2') and (3').

Appendix B: CEBAF-ER Optics

Below are the Optim files showing the optics for 1-pass-up / 1-pass-down operation.

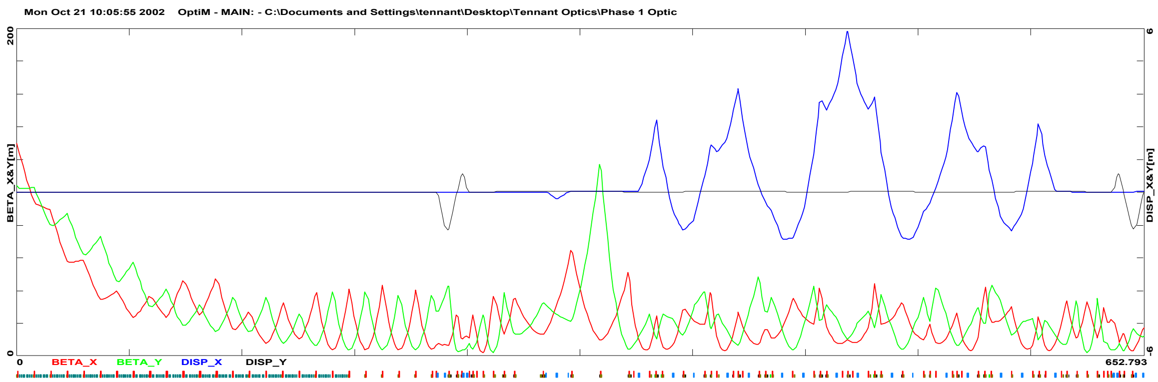


The beam is propagated through the injector, accelerated from 45 MeV to 445 MeV in the north linac and then traverses Arc 1. Note that the beam is matched through the linac.

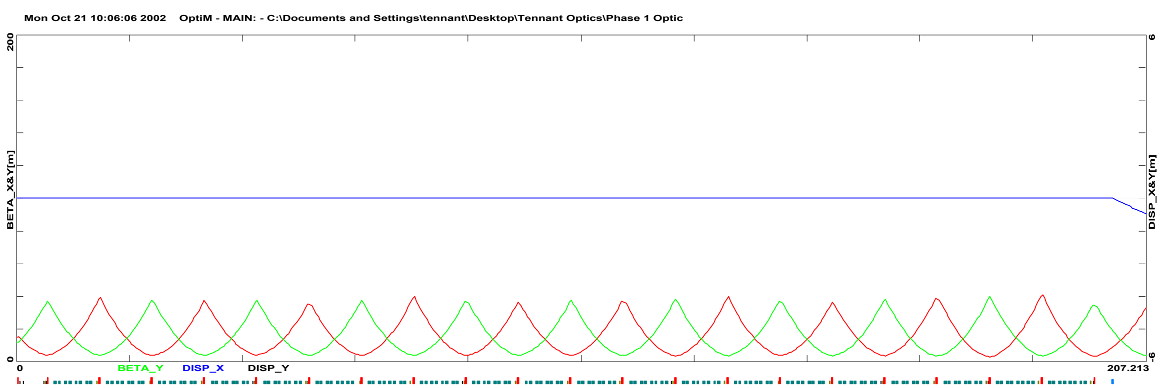


The (mismatched) beam is then accelerated in the south linac from 445 MeV to 845 MeV and using the spreader quads is matched to the Arc 2 optics. The recombiner quads are used to match the optics of the mismatched beam being decelerated through the north linac.

Appendix B: CEBAF-ER Optics (continued)



The mismatched 845 MeV beam is propagated through the north linac and then through Arc 1 as it makes its second pass through the machine.



The beam is then energy-recovered by decelerating through the south linac. The 45 MeV energy-recovered beam is then extracted and sent to the dump.

References

[1] Bogacz A., Benesch J., Butler C., Chao Y., Chattopadhyay S., Dickson R., Douglas D., Guerra A., Hutton, A., Krafft G., Lauze R., May R., Merminga L., Neil G., Oren W., Spata M., Tennant C., Tiefenback M., White K., "CEBAF Energy Recovery Experiment", PAC22 Proposal, 2002

[2] CEBAF-ER Meeting, October 16, 2002

[3] CEBAF-ER Meeting, October 2, 2002

[4] D. Douglas, private communication

[5] Rosenzweig J., Serafini L., *Physical Review E* **49**, vol 2, 1599 (1994)

[6] D. Douglas, private communication