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Accuracy of Prediction of the Multi-Pass Beam Breakup Threshold Current in Beam Transfer Function and "Decay Time" Measurements

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Abstract

The threshold of the multi-pass beam breakup (BBU) instability can be predicted via beam measurements below the threshold. Two measurement techniques include the Beam Transfer Function (BTF) method and the "decay time" method. Assuming a given accuracy of the measurements we estimate the accuracy for the prediction of the threshold current.

Introduction

One can predict the threshold current for BBU by measuring the dependence of the effective quality factor, Q_{eff}, of the beam-plus-HOM system on the average beam current. Details of the measurements themselves (hardware, analysis, etc…) will be left for a future discussion. This note is primarily intended to address the accuracy with which these measurements can predict the threshold current. The effective quality factor can be determined from the width or height of the beam transfer function using the BTF method (a frequency domain measurement). The other method of determining the effective quality factor is a direct measurement of the decay time of oscillations of the beam-plus-HOM system (a time domain measurement). In this paper, we estimate the accuracy of the "decay time" method, assuming a given accuracy of the decay time measurements. First, we consider a case when the average beam current is much smaller than the threshold current. Then, we present the formula for the general case. Everything that is presented here for the "decay time" method is also applicable to the BTF method because both methods are identical in their nature.

I. The Beam Current is Much Smaller than the Threshold Current ($I_b << I_{th}$)

According to [1], the decay time is inversely proportional to the difference between the threshold current and the beam current. To estimate the threshold current we have to make at least two measurements of the decay time:

$$\begin{aligned} \tau_1 &= \frac{\alpha}{I_{th} - I_1} \approx \frac{\alpha}{I_{th}}\left(1 + \frac{I_1}{I_{th}}\right) \\ \tau_2 &= \frac{\alpha}{I_{th} - I_2} \approx \frac{\alpha}{I_{th}}\left(1 + \frac{I_2}{I_{th}}\right) \end{aligned} \tag{1}$$

where α denotes the constant of proportionality. The ratio of the last two equations is

$$\frac{\tau_2}{\tau_1} = \frac{1+\dfrac{I_2}{I_{th}}}{1+\dfrac{I_1}{I_{th}}} \approx \left(1+\frac{I_2}{I_{th}}\right)\left(1-\frac{I_1}{I_{th}}\right) \approx 1+\frac{I_2-I_1}{I_{th}} \tag{2}$$

The last equation yields the threshold current as a function of the ratio τ_2/τ_1:

$$I_{th} = \frac{I_2-I_1}{\left(\dfrac{\tau_2}{\tau_1}-1\right)} \tag{3}$$

The error in the prediction of the threshold current can be expressed through the error in the measurements of the ratio τ_2/τ_1:

$$\delta I_{th} = -\frac{I_2-I_1}{\left(\dfrac{\tau_2}{\tau_1}-1\right)^2}\,\delta\left(\frac{\tau_2}{\tau_1}\right) \tag{4}$$

The error of measurement of the ratio τ_2/τ_1 can be expressed via the error of measurement of the decay time using the formula:

$$\frac{\delta\left(\dfrac{\tau_2}{\tau_1}\right)}{\left(\dfrac{\tau_2}{\tau_1}\right)} = \frac{\delta\tau_1}{\tau_1}+\frac{\delta\tau_2}{\tau_2} = 2\left(\frac{\delta\tau}{\tau}\right) \tag{5}$$

where we assumed that the decay time is always measured with the same relative error $\delta\tau/\tau$. Using (3), (4), and (5) we find that

$$\frac{\delta I_{th}}{I_{th}} = -\frac{2\tau_2}{\tau_2-\tau_1}\left(\frac{\delta\tau}{\tau}\right) \tag{6}$$

Using the exact formula for the decay time (see equation (1)) we can rewrite (6) as

$$\frac{\delta I_{th}}{I_{th}} = -\frac{2\cdot I_{th}}{I_2-I_1}\left(\frac{\delta\tau}{\tau}\right) \tag{7}$$

Formula for the General Case

Here we simply present the formula for a general case without derivation. One can easily derive this formula using equations (1) (the first, exact part of the equation (1)) and (5):

$$\frac{\delta I_{th}}{I_{th}} = -\frac{I_2 - I_1}{\tau_2 I_2 - \tau_1 I_1} \cdot \frac{2\tau_2\tau_1}{\tau_2 - \tau_1}\left(\frac{\delta\tau}{\tau}\right) \tag{8}$$

Discussion

- One can obtain the highest accuracy if the first measurement is done at zero current. Then the decay time τ_1 is equal to $2Q_o/\omega$ where Q_o is the quality factor of the HOM without the beam. It is trivial to measure the Q_o (or τ_o) either directly using a network analyzer or through the "decay time" technique, by abruptly turning off the beam current and observing the HOM voltage decay.

- If the beam current is substantially smaller than the threshold current one can easily conclude from equation (7) that accuracy of time (or Q) measurements has to be much higher than the desirable accuracy of prediction of the threshold current. For example, let's assume that I_1 and I_2 are equal to 5% and 10% of the threshold current, respectively. Assume that the accuracy of time measurements is 1%. Using equation (7) we get an accuracy of prediction of the threshold current of 40%:

$$\frac{\delta I_{th}}{I_{th}} = -\frac{2}{0.1 - 0.05}0.01 = 0.4\,. \tag{8}$$

References

[1] E. Pozdeyev, C. Tennant, "Equation for the Multi-Pass Beam Breakup Threshold Current for a Single Mode and a 4x4 Recirculation Matrix", JLAB-TN-019, (2004).