A NOTE ON MULTI-PASS BEAM BREAKUP WITH A COUPLED OPTICS

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We study a multi-pass beam breakup in a recirculating accelerator when transverse beam optics is coupled. We find that the reason for apparent enhancement of beam breakup threshold observed at the JLAB IR Upgrade is not the one being discussed so far.

INTRODUCTION

Recently a multi-pass beam breakup was observed at JLAB IR FEL Upgrade. As a part of an effort to operate machine at a current beyond the observed beam breakup threshold a rotator was introduced to couple x and y beam motion intentionally. The idea behind this is to break a chain of a feedback mechanism by a rotator turning an x impulse to y offset rather than to a x offset which then could then further excite the wake which caused the x impulse initially. However, one should note that a dipole mode has always two polarizations. Unless one eliminates the other polarization completely, which is impossible in practice, a chain of a feedback mechanism will be re-established through coupled beam motions. We have studied this chain of beam motions and found a beam breakup threshold formula which should be a useful guide in understanding this rotator method of overcoming the beam breakup problem in a recirculating accelerator

A BBU THRESHOLD FORMULA

In the framework of a simplified one cavity and one recirculation model of an accelerator (which has proved to be a surprisingly accurate approximation for the JLAB IR FEL Upgrade where there exists a single dominant dipole mode causing a beam breakup) we have studied BBU with a coupled beam motion.

To prove our point mentioned in the Introduction we present the of a single mode with two polarizations of equal strength here. A general case is quite a bit more complicated computationally. The following beam breakup threshold formula is valid when M_{12} and M_{34} are all made to vanish (transfer matrix is in DIMAD units) which can be arranged with a rotator as shown nicely by D. Douglas for the JLAB IR FEL Upgrade:

$$I_{th} = \frac{2E\omega \exp\left(-\frac{\omega t_\tau}{2Q}\right)}{ec\left(\frac{Z'' T^2}{Q}\right)Q\sqrt{M_{14}M_{32}}\left|\sin \omega t_\tau\right|}$$

for $M_{14}\, M_{32} > 0$
and

$$I_{th} = \frac{2E\omega \exp\left(-\frac{\omega t_\tau}{2Q}\right)}{ec\left(\frac{Z''T^2}{Q}\right)Q\sqrt{-M_{14}M_{32}}\left|\cos\omega t_\tau\right|}$$

for $M_{14}\,M_{32} < 0$

Our notations are:
e = elctron charge
c = velocity of light
E = beam energy
t_τ = recirculation period
ω = dipole mode frequency
Q = quality factor of the mode
$(Z''T^2/Q)$ = shunt impedance of the mode

Note that beam breakup threshold current now depends on the absolute value of phase lag terms sin ωt_τ or cos ωt_τ and is not likely to become arbitrarily large in no circumstance even at this lowest order of approximation.

Mathematical details will not be presented here because they are not particularly interesting. But the above formula is. We also note that a different formula can be obtained when x and y polarizations are unsymmetrical in strength and numerical bench marking is a next step.