

A SINGLE CAVITY MODEL OF MULTI-PASS BEAM BREAKUP IN RECIRCULATING LINAC

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We investigate a multi-pass beam breakup in a recirculating accelerator in the framework of a single cavity model of the accelerator. We derive approximate but fairly accurate expressions for the beam breakup threshold current for various situations. These formulae should serve as a guide to understand BBU phenomenon for a particular system and also as a tool to estimate the BBU threshold current quickly.

NOTATIONS

e = electron charge
 c = velocity of light
 E = beam energy
 t_τ = recirculation period
 E = beam energy
 E_i = beam energy at the i -th pass
 t_τ = recirculation time
 t_τ^{ij} = recirculation time from the i -th pass to the j -th pass
 t_0 = bunching period
 ω_0 = bunching frequency
 ω = dipole mode frequency
 ω_x = x-polarized mode frequency
 ω_y = y-polarized mode frequency
 Q = quality factor of the mode
 Q_x = quality factor of x-mode
 Q_y = quality factor of y-mode
 $(Z''T^2/Q)$ = shunt impedance of the mode
 $(Z''T^2/Q)_x$ = shunt impedance of x-polarized mode
 $(Z''T^2/Q)_y$ = shunt impedance of y-polarized mode
 M^{ij} = beam transfer matrix from the i -th pass to the j -th pass

A TWO PASS MACHINE: TRANSVERSE BEAM OPTICS IS SEPARATED

A single cavity with one dominating dipole mode model of an accelerator has proved to be a surprisingly accurate approximation for JLAB IR FEL Upgrade. With no coupling in transverse beam motion this is the simplest case to handle.

A 1st order formula:

$$I_{th} = - \frac{2E\omega}{ec \left(\frac{Z'' T^2}{Q} \right) Q M_{12} \sin \omega t_\tau}$$

A '2nd' order formula:

$$I_{th} = \frac{2E \left(\sin \omega t_\tau \mu \sqrt{\sin^2 \omega t_\tau + \frac{2\omega}{Q} \Delta} \right)}{ec \left(\frac{Z'' T^2}{Q} \right) M_{12} \Delta}$$

where

$$\Delta = \frac{t_0}{2} \{ (1 - \sin \omega t_0) \cos 2\omega t_\tau - \sin \omega (2t_\tau + t_0) \} + t_\tau \cos^2 \omega t_\tau$$

For a y-polarization M_{12} should be replaced by M_{34} .

AN N>2 PASS MACHINE: TRANSVERSE BEAM OPTICS IS SEPARATED

For more than one recirculation relevant machine parameters needed to be considered quickly multiplies. However, the following BBU threshold expression should provide some guidance in analyzing BBU problem for a particular machine setup especially in the designing stage.

$$I_{th} = - \frac{2E\omega}{ec \left(\frac{Z'' T^2}{Q} \right) Q \sum_{j>i=1}^N \frac{E}{E_i} (M^{ij})_{12} \sin \omega t_\tau^{ij}}$$

For a y-polarization $(M^{ij})_{12}$ should be replaced by $(M^{ij})_{34}$.

A TWO PASS MACHINE: TRANSVERSE BEAM OPTICS IS COUPLED

1. A SYMMETRIC CASE

We assume a single dipole mode with two polarizations of identical strength causing BBU. We also assume $M_{13} = M_{34} = 0$ with a rotator as has been demonstrated at JLAB IR FEL Upgrade. We find

$$I_{th} = \frac{2E\omega}{ec \left(\frac{Z'' T^2}{Q} \right) Q \sqrt{M_{14} M_{32}} |\sin \omega t_\tau|}$$

for $M_{14} M_{32} > 0$
and

$$I_{th} = \frac{2E\omega}{ec \left(\frac{Z'' T^2}{Q} \right) Q \sqrt{-M_{14} M_{32}} |\cos \omega t_\tau|}$$

for $M_{14} M_{32} < 0$

A TWO PASS MACHINE: TRANSVERSE BEAM OPTICS IS COUPLED

II. AN ASYMMETRIC CASE

In the case of two unsymmetrical polarizations new features appear reflecting the change in physics of driven coupled resonators. Assuming that the x-polarization is the dominating one we obtain the following BBU threshold current expression:

$$I_{th} = \sqrt{\frac{2}{\pi}} \frac{E \sqrt{\omega_0 \omega_x} \exp \left[\frac{1}{2} \left(\frac{\omega_y}{2Q_y} - \frac{\omega_x}{2Q_x} \right) t_0 \right]}{ec \sqrt{\left(\frac{Z'' T^2}{Q} \right)_y \left(\frac{Z'' T^2}{Q} \right)_x} Q_x \sqrt{-M_{14} M_{32}} \sin \omega_x (2t_\tau + t_0) \sin \omega_y t_0}$$

Note that the bunching frequency plays a prominent role in determining the threshold current.

A TWO PASS MACHINE: TRANSVERSE BEAM OPTICS IS COUPLED

III. A ROTATED POLARIZATION

We assume a single dipole mode with two polarizations of identical strength causing BBU to make the problem manageable algebraically. The x-polarization (horizontal in our convention) is now rotated by an angle α .

Let

$$M^2 \equiv (M_{12} - M_{34})^2 + 4M_{14}M_{32}$$

Case $M^2 = 0$:

$$I_{th} = -\frac{4E\omega}{ec\left(\frac{Z''T^2}{Q}\right)Q(M_{12} + M_{34})\sin\omega t_\tau}$$

Case $M^2 > 0$: the threshold current is the smaller of two values.

$$I_{th} = -\frac{4E\omega}{ec\left(\frac{Z''T^2}{Q}\right)Q(M_{12} + M_{34} \pm M)\sin\omega t_\tau}$$

Case $M^2 < 0$: the threshold current is the smaller of two values.

$$I_{th} = -\frac{4E\omega}{ec\left(\frac{Z''T^2}{Q}\right)Q\{(M_{12} + M_{34})\sin\omega t_\tau \pm |M|\cos\omega t_\tau\}}$$

Note that I_{th} does not depend on the rotation angle of polarization in this case. But we also should mention that BBU threshold expressions will contain A , B , C and D defined in below when two polarizations are not completely degenerate as treated here. These quantities now have an explicit dependency on the rotation angle α :

$$\begin{aligned} A &= M_{12} \cos^2 \alpha + M_{34} \sin^2 \alpha + (M_{14} + M_{32}) \cos \alpha \sin \alpha \\ B &= M_{14} \cos^2 \alpha - M_{32} \sin^2 \alpha + (M_{34} - M_{12}) \cos \alpha \sin \alpha \\ C &= M_{32} \cos^2 \alpha - M_{14} \sin^2 \alpha + (M_{34} - M_{12}) \cos \alpha \sin \alpha \\ D &= M_{34} \cos^2 \alpha + M_{12} \sin^2 \alpha - (M_{14} + M_{32}) \cos \alpha \sin \alpha \end{aligned}$$

A THREE PASS MACHINE WITH BEAM OPTICS ROTATED BY 90 DEGREE WITH A ROTATOR

We assume a single dipole mode with two polarizations of identical strength and also assume the following form of beam transfer matrices.

$$M^{12} = \begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix} \quad \text{and} \quad M^{23} = \begin{pmatrix} C & 0 \\ 0 & D \end{pmatrix}$$

We find the following expression for the BBU threshold in this case:

$$I_{th} = - \frac{2E\omega}{ec \left(\frac{Z'' T^2}{Q} \right) Q \{ \text{Re } M_{\pm} \cdot \sin \omega t_{\tau}^{23} + \text{Im } M_{\pm} \cdot \cos \omega t_{\tau}^{23} \}}$$

The threshold current is determined by the smaller of two values. In the formula $\text{Re}M_{\pm}$ and $\text{Im}M_{\pm}$ has been defined as below:

$$\text{Re } M_{\pm} + i \text{Im } M_{\pm} = \frac{M_{12}^{23} + M_{34}^{23}}{2} \pm \left[\frac{1}{4} (M_{12}^{23} - M_{34}^{23})^2 + \left\{ M_{14}^{12} M_{32}^{12} e^{2i\omega(t_{\tau}^{12} - t_{\tau}^{23})} + (M_{14}^{12} M_{32}^{13} + M_{32}^{12} M_{14}^{13}) \cdot \left\{ e^{i\omega(2t_{\tau}^{12} - t_{\tau}^{23})} + M_{14}^{23} M_{32}^{13} e^{2i\omega t_{\tau}^{12}} \right\} \right\} \right]^{1/2}$$

The dependency of the BBU threshold current on various beam transport parameters is already quite complex to analyze. Obviously, an analytic solution for more general cases than the one presented here would be hard to obtain and a relation obtained is not expected to be much superior to a numerical simulation in understanding BBU phenomenon, either. Therefore, I stop here.