Beam Induced HOM Power Spectrum in JLab 1MW ERL-FEL*

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A high current (1A class, averaged) cryomodule design in a superconducting Energy Recovered Linac (ERL) based Free Electron Laser (FEL) requires a very careful cavity shape choice and moderated Higher Order Mode (HOM) damping and cooling scheme to handle a large amount of beam induced resonance or average power. Such a power spectrum depends on the beam time structure (bunch charge, length, photo-cathode laser frequency in CW or pulsed mode and beam recirculation time) and cavity's impedance (HOM frequency, R/Q and external Q). A time–averaged HOM power spectrum normalized to the cavity's R/Q has been calculated by Sangho Kim for the Spallation Neutron Source (SNS) beam time structure (single pass). This calculation method has been confirmed for the SNS case and modified to calculate the HOM power for the ERL-FEL case. A MathCAD program has been written to evaluate four scenarios. (1): 1A, 750MHz CW beam; (2): 0.1A, 75MHz CW beam; (3): case (2) with a different beam recirculation time; and (4): 75MHz pulsed beam. These results have been given a guideline to the design of a 1A class RF cavity and cryomodule.

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Time Structure of SNS Beam & HOM Induced Voltage Analysis Based on Sangho Kim's SNS/AP Technical Note No.10 (also in NIM in Physical Research A 492 (2002)1-10)

Haipeng Wang, Last updated on April. 28, 2005

HOM decay time constant: $T_d(Q_{ex}, \omega) := 2 \cdot \frac{Q_{ex}}{\omega}$ (s)

speed of light: $c := 299792458$ (m/s)

SRF cavity's fundamental mode frequency: $f_1 := 805 \cdot 10^6$ (Hz)

Total numbers of Micro-pulses within one Midi-pulse: $N := 260$

Total numbers of Midi-pulses within one Macro-pulse: $M := 1060$

Pulse period between each Micro-pulse: $T_b := \frac{2}{f_1}$

(=1/402.5MHz=2/805MHz)

$T_b = 2.48447205 \times 10^{-9}$ (s)

Bunch period between each Midi-pulse: $T_i := T_b \cdot 380$

(=1/1.059MHz) $T_i = 9.44099379 \times 10^{-7}$ (s)

Bunch period between each Macro-pulse: $T_m := \frac{1}{60}$ (s)

$T_m = 0.01666667$ (s)

Macro-pulse length: $T_{mb} := T_i \cdot 1060$ (s)

$T_{mb} = 1.00074534 \times 10^{-3}$

Macro-pulse spacing: $T_G := T_m - T_{mb}$ $T_G = 0.01566592$ (s)

Midi-pulse length: $T_{ib} := T_b \cdot 260$ (s)

Midi-pulse spacing: $T_g := T_i - T_{ib}$ $T_g = 2.98136646 \times 10^{-7}$ (s)

averaged macro-pulse beam current: $I_{ma} := 26 \cdot 10^{-3}$ (A)

single H- ion bunch charge (approximate to a point charge): $q := \frac{I_{ma} \cdot T_{mb}}{N \cdot M}$ (C)

$q = 9.44099379 \times 10^{-11}$ (C)

R/Q normalized induced voltage by a point charge: $V_q(\omega) := \frac{\omega}{2} \cdot q$ (V)

single bunch length 8° of 805MHz: $T_p := \frac{8}{360} \cdot \frac{1}{f_1}$ $T_p = 2.761 \times 10^{-11}$ (s)

Micro-pulse or between single bunch spacing: $T_{bp} := T_b - T_p$ $T_{bp} = 2.457 \times 10^{-9}$ (s)

At the time just after kth macro-pulse:

$$A(k, Q_{ex}, \omega) := V_q(\omega) \cdot \left(\frac{1 - \exp\left(\frac{-N \cdot T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot N \cdot T_b \right)}{1 - \exp\left(\frac{-T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_b \right)} \right) \cdot \left(\frac{1 - \exp\left(\frac{-M \cdot T_i}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot M \cdot T_i \right)}{1 - \exp\left(\frac{-T_i}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_i \right)} \right) \cdot \left(\frac{1 - \exp\left(\frac{-k \cdot T_m}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot k \cdot T_m \right)}{1 - \exp\left(\frac{-T_m}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_m \right)} \right)$$

During the gap between kth and (k+1)th Macro-pulses:

$$B(k, Q_{ex}, \omega, t_1) := A(k, Q_{ex}, \omega) \exp\left(\frac{-t_1}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot t_1\right)$$

At the time just after mth Midi-pulse in (k+l)th Macro-pulse (please note the difference here from Sang-ho's original that has typo errors) :

$$C(m, k, Q_{ex}, \omega) := B(k, Q_{ex}, \omega, T_G) \cdot \exp\left[\frac{-m \cdot T_i + T_g}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot (m \cdot T_i - T_g)\right] + V_q(\omega) \cdot \left(\frac{1 - \exp\left(\frac{-N \cdot T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot N \cdot T_b\right)}{1 - \exp\left(\frac{-T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_b\right)} \cdot \frac{1 - \exp\left(\frac{-m \cdot T_i}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot m \cdot T_i\right)}{1 - \exp\left(\frac{-T_i}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_i\right)}\right)$$

During the gap between mth and (m+1)th Midi-pulse in (k+1)th Macro-pulse:

$$D(m, k, Q_{ex}, \omega, t_2) := C(m, k, Q_{ex}, \omega) \cdot \exp\left(\frac{-t_2}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot t_2\right)$$

At the time just after nth Micro-pulse in (m+1)th Midi-pulse in (k+1)th Macro-pulse (please note the difference here from Sang-ho's original that has typo errors):

$$E(n, m, k, Q_{ex}, \omega) := D(m, k, Q_{ex}, \omega, T_g) \cdot \exp\left[\frac{-(n-1) \cdot T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot (n-1) \cdot T_b\right] + V_q(\omega) \cdot \left(\frac{1 - \exp\left(\frac{-n \cdot T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot n \cdot T_b\right)}{1 - \exp\left(\frac{-T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_b\right)}\right)$$

During the gap between nth and (n+1)th Micro-pulses in (m+1)th Midi-pulse in (k+1)th Macro-pulse:

$$F(n, m, k, Q_{ex}, \omega, t_3) := E(n, m, k, Q_{ex}, \omega) \cdot \exp\left(\frac{-t_3}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot t_3\right)$$

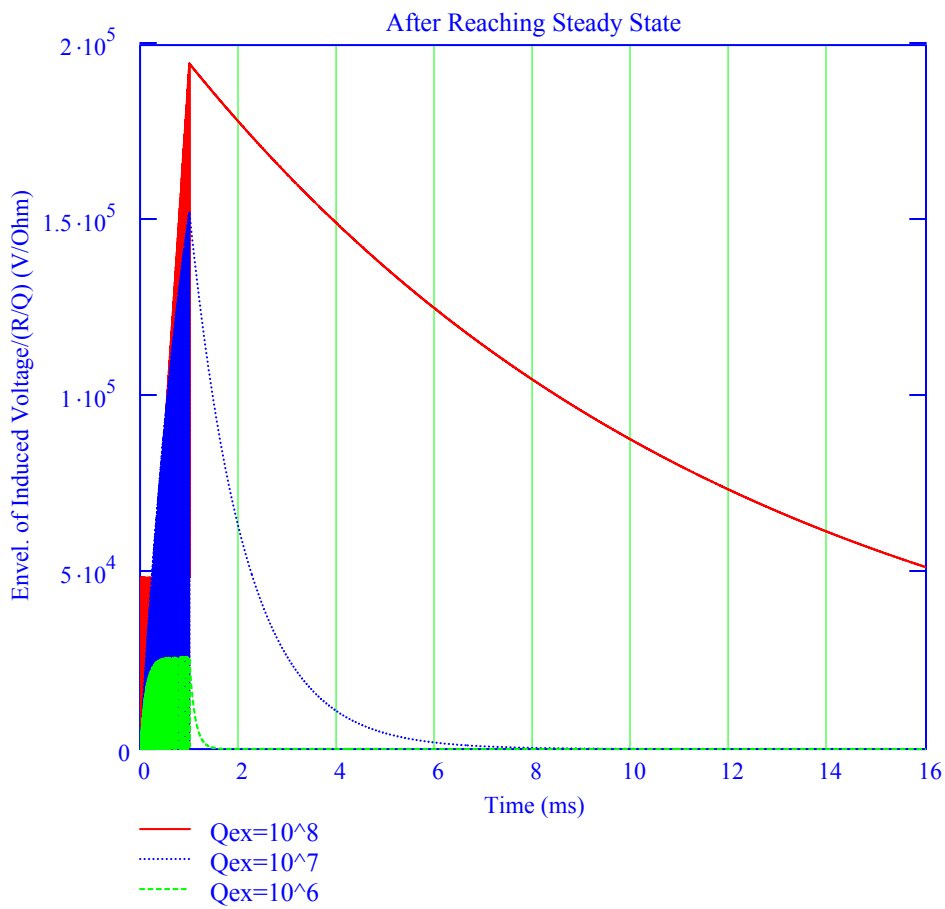
Pulsed beam bunches induced voltage can be expressed as:

$$
\begin{aligned}
V(Q_{ex}, \omega, t) := \;& t_{11} \leftarrow \mathrm{mod}(t, T_m) \\
& \text{if } t_{11} \geq T_{mb} \\
& \quad k \leftarrow \mathrm{ceil}\left(\frac{t}{T_m}\right) \\
& \quad t_1 \leftarrow \mathrm{mod}(t, T_m) - T_{mb} \\
& \quad B(k + 3000, Q_{ex}, \omega, t_1) \\
& \text{if } t_{11} < T_{mb} \\
& \quad t_{22} \leftarrow \mathrm{mod}(t_{11}, T_i) \\
& \quad \text{if } t_{22} \geq T_{ib} \\
& \quad\quad m \leftarrow \mathrm{ceil}\left(\frac{t_{11}}{T_i}\right) \\
& \quad\quad t_2 \leftarrow \mathrm{mod}(t_{11}, T_i) - T_{ib} \\
& \quad\quad D(m, k + 3000, Q_{ex}, \omega, t_2) \\
& \quad \text{if } t_{22} < T_{ib} \\
& \quad\quad t_{33} \leftarrow \mathrm{mod}(t_{22}, T_b) \\
& \quad\quad \text{if } t_{33} \geq T_p \\
& \quad\quad\quad n \leftarrow \mathrm{ceil}\left(\frac{t_{22}}{T_b}\right) \\
& \quad\quad\quad t_3 \leftarrow \mathrm{mod}(t_{22}, T_b) - T_p \\
& \quad\quad\quad F(n, m, k + 3000, Q_{ex}, \omega, t_3) \\
& \quad\quad V_q(\omega) \;\; \text{if } t_{33} < T_p
\end{aligned}
$$

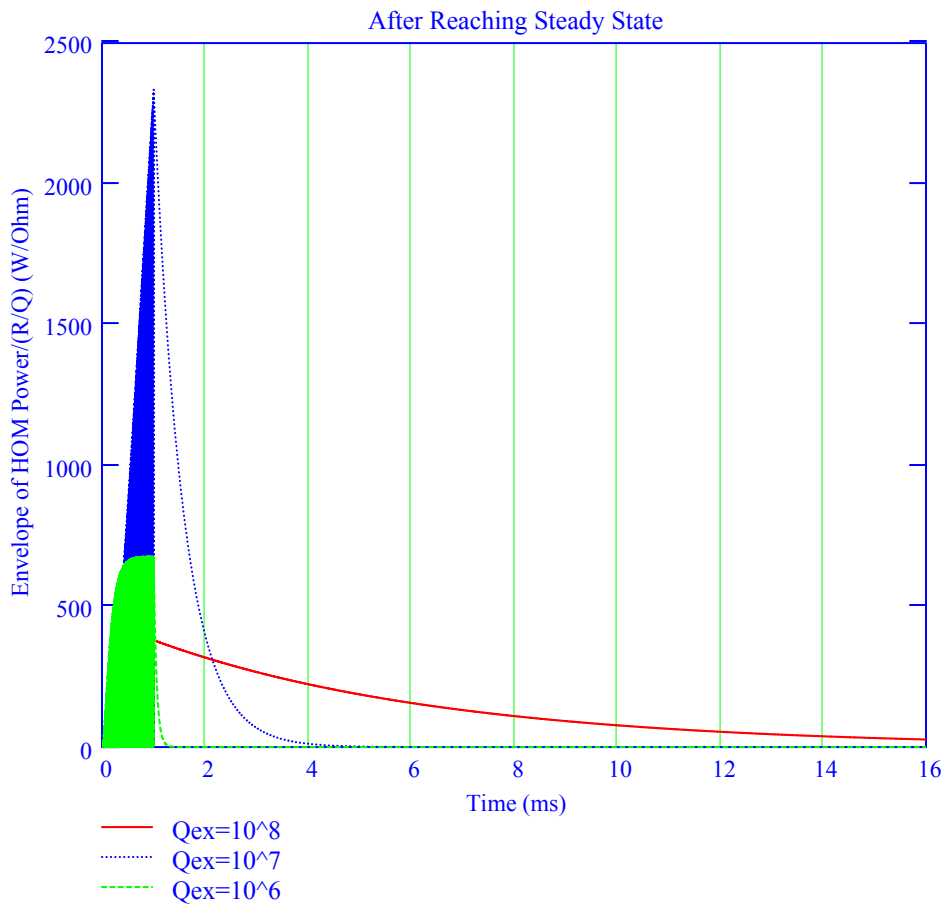
The HOM power dissipated by the beam is:

$$P(Q_{ex},\omega,t) := \frac{\left(\left|V(Q_{ex},\omega,t)\right|\right)^2}{Q_{ex}} \qquad t := 0, 1.1\cdot 10^{-6} .. 16\cdot 10^{-3} \quad \text{(s)} \qquad f := 2.8175\cdot 10^{9} \quad \text{(Hz)} \qquad \omega := 2\cdot\pi\cdot f$$

Plot of envelop of the induced voltage/(R/Q) at f=2.8175 GHz (7*402.5 MHz) between the macro-pulses in steady state:



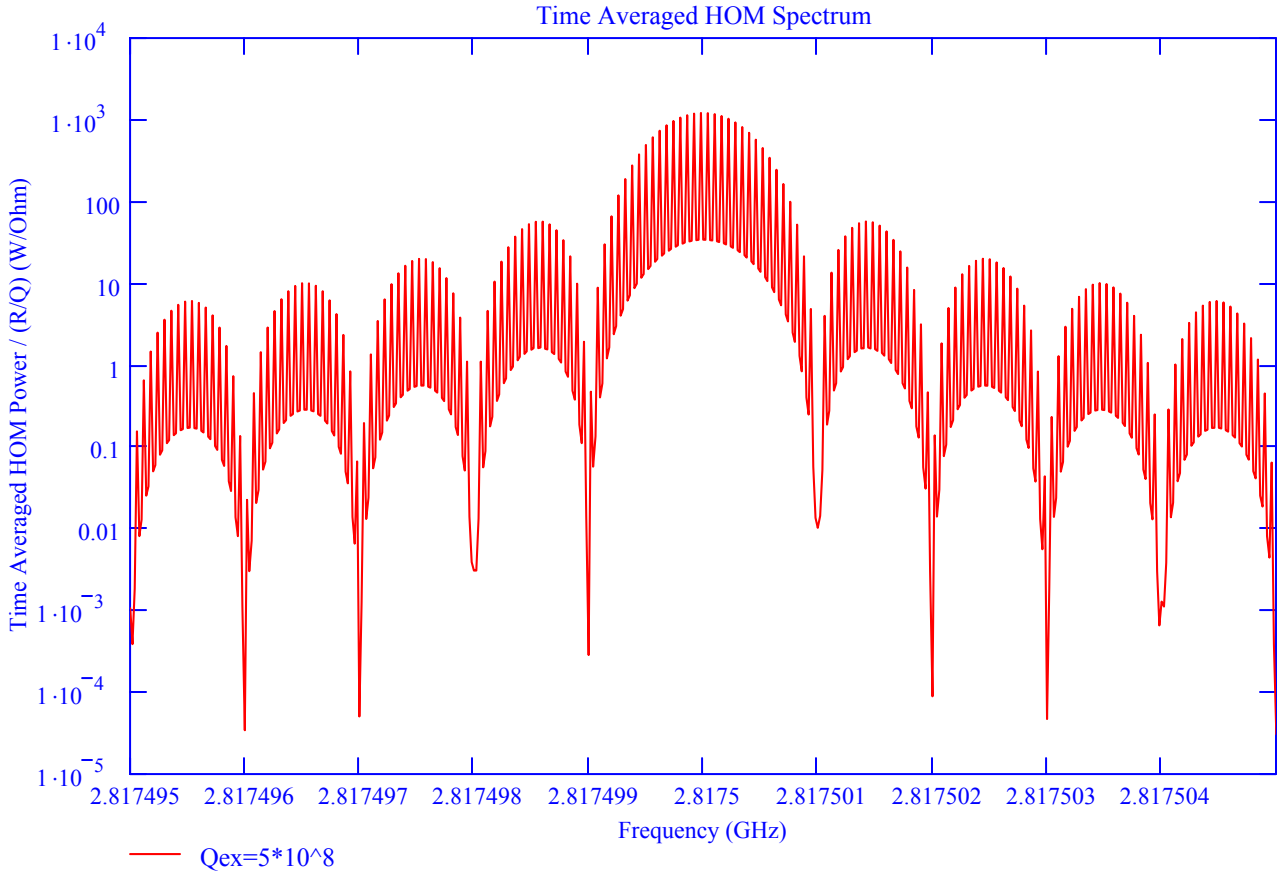
Plot of envelop of HOM power/(R/Q) at f=2.8175 GHz (7*402.5 MHz) between the macro-pulses in steady state:



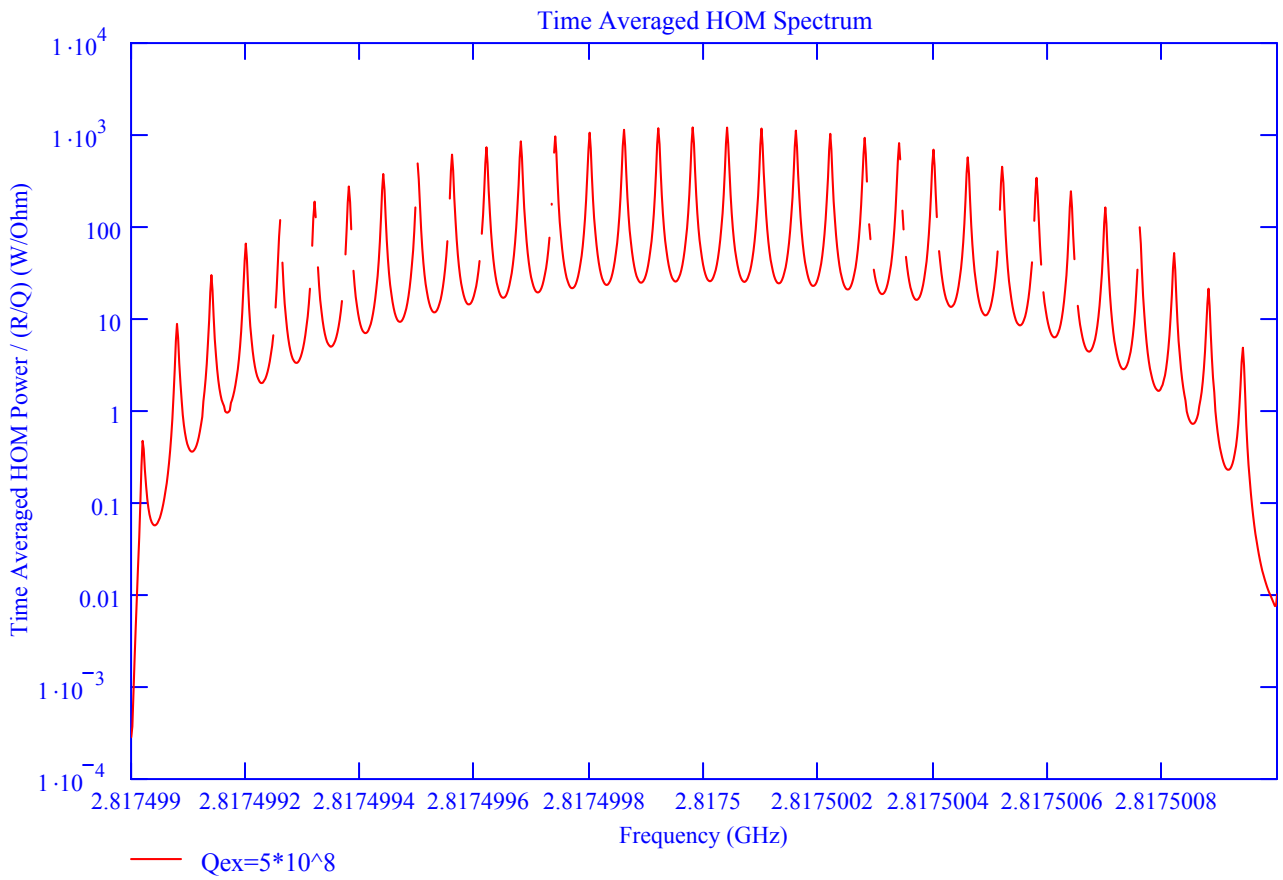
The time averaged HOM power can be obtained by integrating the $P(Q_{ex},\omega,t)$ in one period of T_m:

$$P_{ave}(Q_{ex},\omega) := \frac{1}{T_m}\cdot\int_0^{T_m} P(Q_{ex},\omega,t)\,dt$$

$$\omega := 2\cdot\pi\cdot\left(2.8175\cdot10^9 - 5\cdot10^3\right), 2\cdot\pi\cdot\left(2.8175\cdot10^9 - 5\cdot10^3 + 20\right) .. 2\cdot\pi\cdot\left(2.8175\cdot10^9 + 5\cdot10^3\right)$$



$$\omega := 2\cdot\pi\cdot\left(2.8175\cdot 10^{9} - 1\cdot 10^{3}\right), 2\cdot\pi\cdot\left(2.8175\cdot 10^{9} - 1\cdot 10^{3} + 2\right) .. 2\cdot\pi\cdot\left(2.8175\cdot 10^{9} + 1\cdot 10^{3}\right)$$



Time Structure of JLab FEL Upgrade (1A, CW) Beam & HOM Induced Voltage Analysis Based on Sangho Kim's SNS/AP Technical Note No.10 and Correction from His Mathematica File

Haipeng Wang, Last updated on April 28, 2005

HOM decay time constant: $T_d(Q_{ex}, \omega) := 2 \cdot \frac{Q_{ex}}{\omega}$ (s)

speed of light: $c := 299792458$ (m/s)

SRF cavity's fundamental mode frequency: $f_1 := 750 \cdot 10^6$ (Hz)

bunch period between accelerated bunch and decelerated bunch : $T_b := \frac{1}{2 \cdot f_1}$ $T_b = 6.66666667 \times 10^{-10}$ (s)

(=1/2/750MHz)

averaged beam current for accelerated beam: $I_{ma} := 1.0$ (A)

electron charge per bunch (approximate to a point charge): $q := I_{ma} \cdot 2 \cdot T_b$ (C) $q = 1.33333333 \times 10^{-9}$ (C)

R/Q normalized induced voltage by a point charge: $V_q(\omega) := \frac{\omega}{2} \cdot q$ (V)

single bunch length 4° of 750MHz: $T_p := \frac{4}{360} \cdot \frac{1}{f_1}$ $T_p = 1.481 \times 10^{-11}$ (s)

bunch spacing between accelerated bunch and decelerated bunch: $T_{bp} := T_b - T_p$ $T_{bp} = 6.519 \times 10^{-10}$ (s)

Beam induced voltage at the time just after nth bunches:

$$A(n, Q_{ex}, \omega) := V_q(\omega) \cdot \left(\frac{1 - \exp\left(\frac{-n \cdot T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot n \cdot T_b \right)}{1 - \exp\left(\frac{-T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_b \right)} \right)$$

Beam induced voltage during the gap between nth and (n+1)th bunches:

$$B(n, Q_{ex}, \omega, t_1) := A(n, Q_{ex}, \omega) \exp\left(\frac{-t_1}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot t_1 \right)$$

Pulsed beam bunches induced voltage can be expressed as:

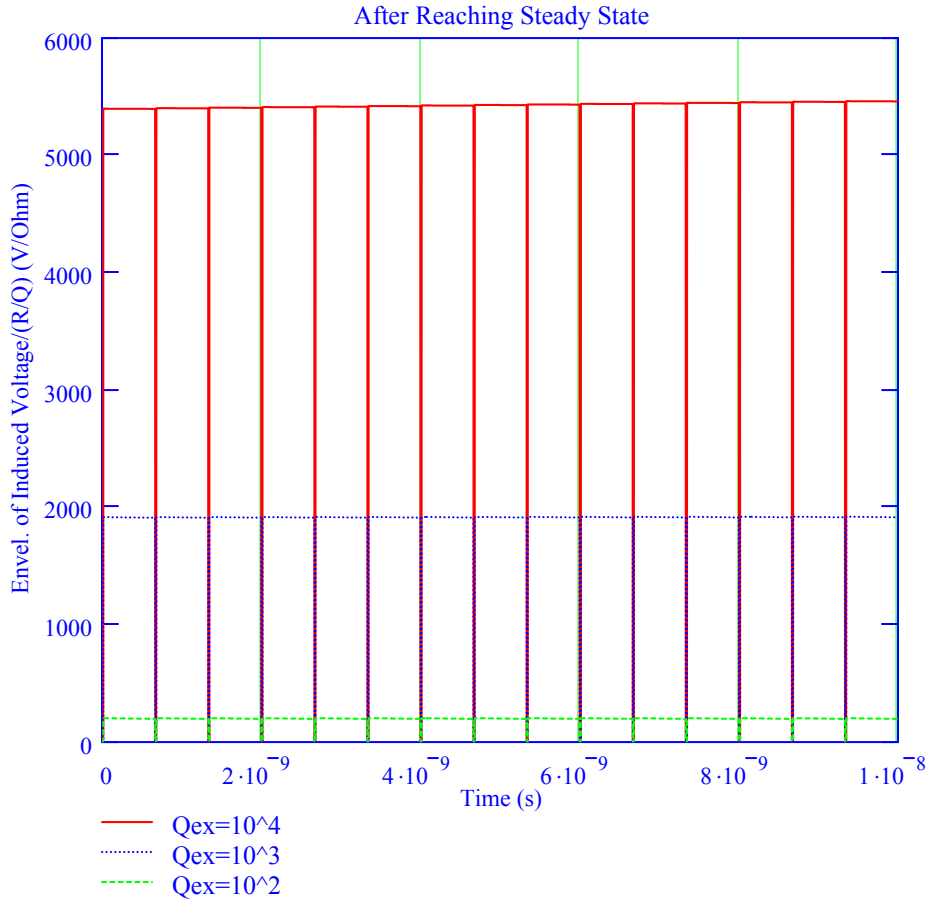
$$V(Q_{ex}, \omega, t) := \left| \begin{array}{l} t_{11} \leftarrow \mathrm{mod}(t, T_b) \\ \text{if } t_{11} \geq T_p \\ \quad \left| \begin{array}{l} n \leftarrow \mathrm{ceil}\left(\frac{t}{T_b} \right) \\ t_1 \leftarrow \mathrm{mod}(t, T_b) - T_p \\ B(n + 1000, Q_{ex}, \omega, t_1) \end{array} \right. \\ V_q(\omega) \quad \text{if } t_{11} < T_p \end{array} \right.$$

The HOM power dissipated by the beam is:

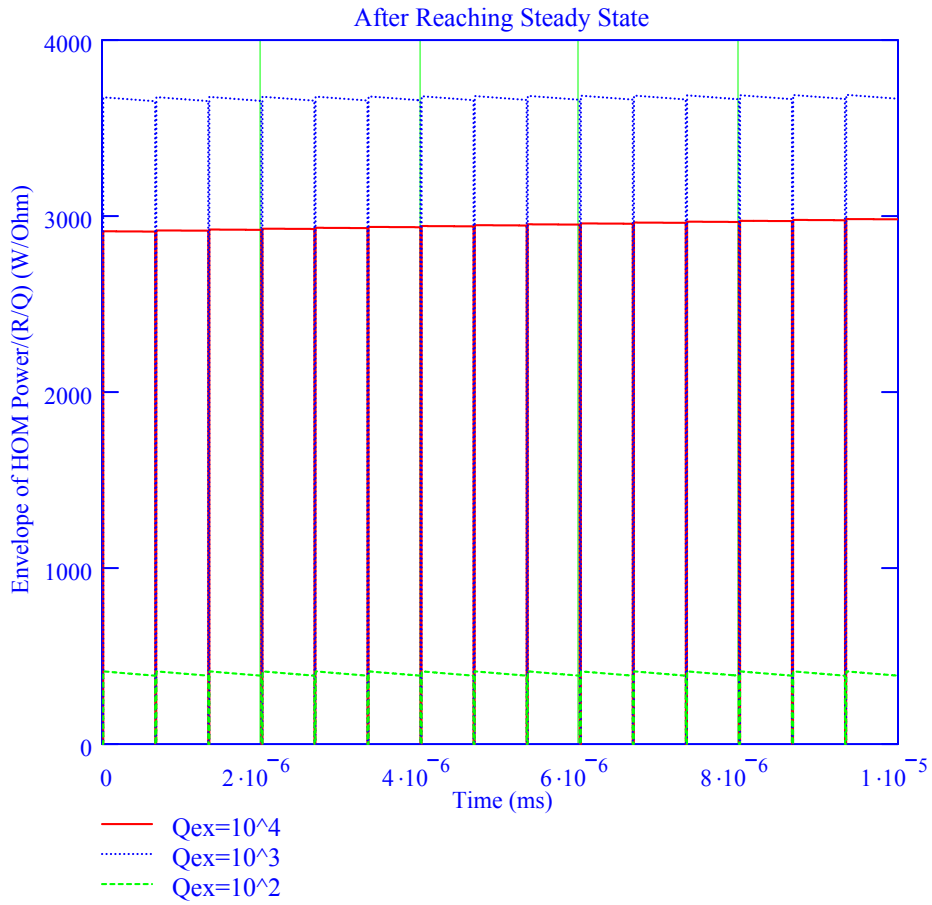
$$P(Q_{ex}, \omega, t) := \frac{\left(\left| V(Q_{ex}, \omega, t) \right| \right)^2}{Q_{ex}}$$

$t := 0, 1.1 \cdot 10^{-12} .. 10 \cdot 10^{-9}$ (s) $f := 1500 \cdot 10^6$ (Hz) $\omega := 2 \cdot \pi \cdot f$

Plot of envelop of the induced voltage/(R/Q) at f=1.5 GHz (2*750 MHz) after 1000 bunches nearly in steady state:



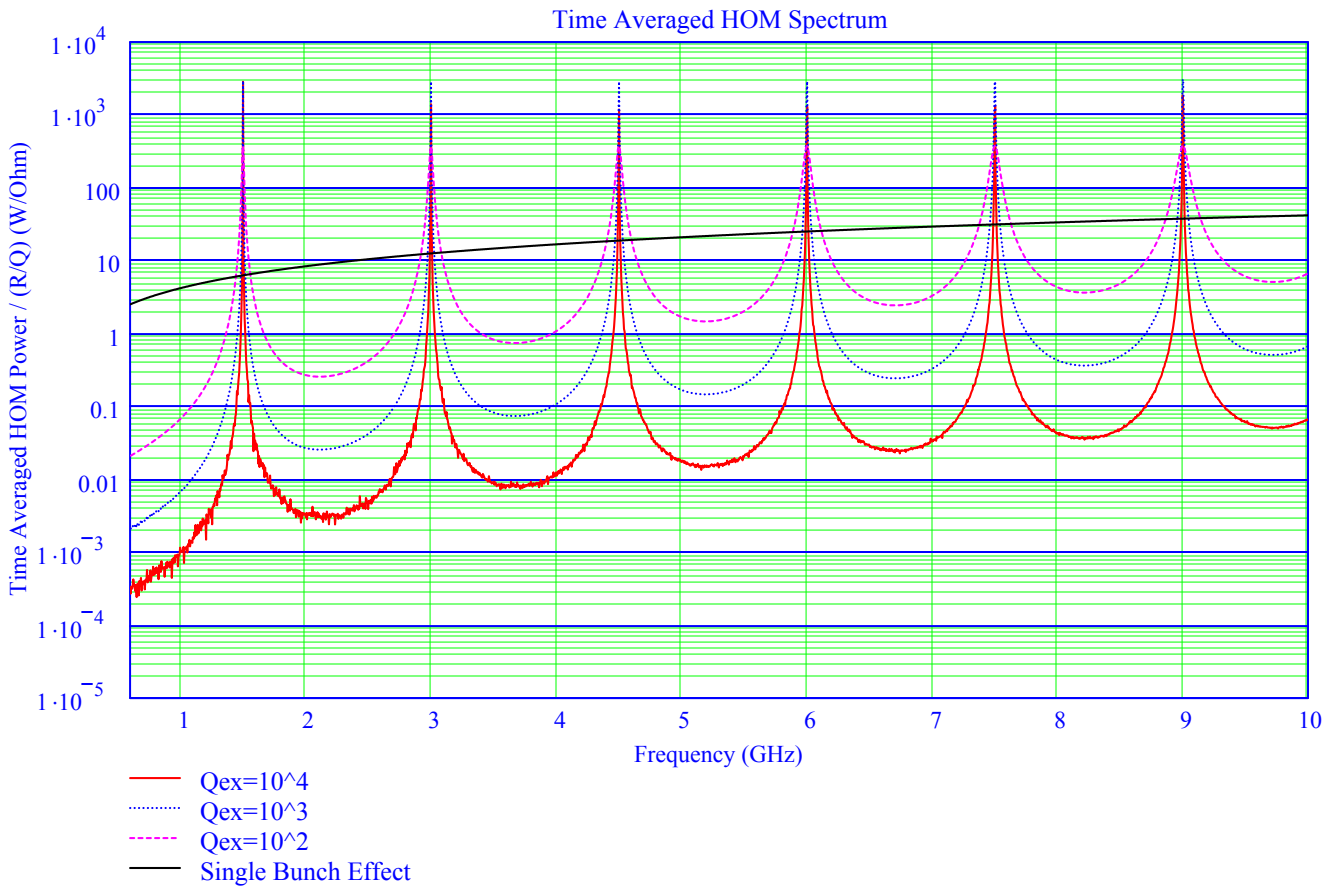
Plot of envelop of HOM power/(R/Q) at f=1.5 GHz (2*750 MHz) after 1000 bunches nearly in steady state:



The time averaged HOM power can be obtained by integrating the $P(Q_{ex},\omega,t)$ in 1000 periods of T_b:

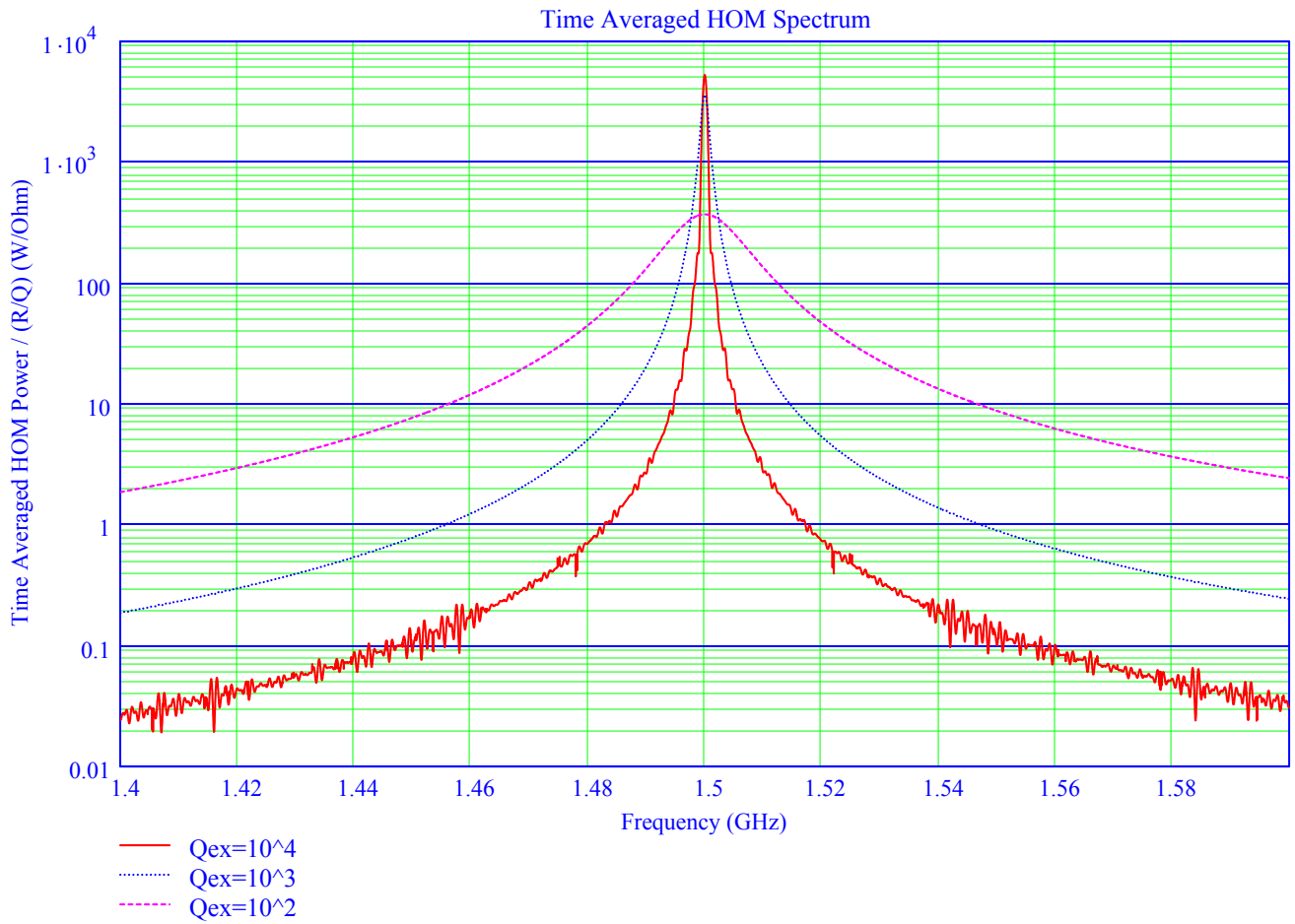
$$P_{ave}(Q_{ex},\omega) := \frac{1}{1000\cdot T_b}\cdot\int_0^{1000\cdot T_b} P(Q_{ex},\omega,t)\,dt$$

$$\omega := 2\cdot\pi\cdot\left(5\cdot10^9 - 5\cdot10^9\right), 2\cdot\pi\cdot\left(5\cdot10^9 - 5\cdot10^9 + 4.4\cdot10^6\right) .. 2\cdot\pi\cdot\left(5\cdot10^9 + 5\cdot10^9\right)$$



Zoom-in above picture around 1.5GHz peak with finer resolution:

$$\omega := 2\cdot\pi\cdot\left(1.5\cdot10^{9} - 0.1\cdot10^{9}\right), 2\cdot\pi\cdot\left(1.5\cdot10^{9} - 0.1\cdot10^{9} + 2\cdot10^{4}\right) .. 2\cdot\pi\cdot\left(1.5\cdot10^{9} + 0.1\cdot10^{9}\right)$$



Time Structure of JLab FEL Upgrade (100mA, CW) Beam & HOM Induced Voltage Analysis Based on Sangho Kim's SNS/AP Technical Note No.10 and Correction from His Mathematica File

Haipeng Wang, Last updated on April 28, 2005

HOM decay time constant: $T_d(Q_{ex}, \omega) := 2 \cdot \frac{Q_{ex}}{\omega}$ (s)

speed of light: $c := 299792458$ (m/s)

SRF cavity's fundamental mode frequency: $f_1 := 750 \cdot 10^6$ (Hz)

photocathode drive laser frequency: $f_2 := 75 \cdot 10^6$ (Hz)

interger number of recirculated path length / fundemantal RF wavelength: $N := 645$

recirculated beam time period: $T_c := \frac{N + \frac{1}{2}}{f_1}$ $T_c = 8.607 \times 10^{-7}$ (s)

bunch period between the accelerated bunches: $T_a := \frac{1}{f_2}$ $T_a = 1.33333333 \times 10^{-8}$ (s)

minor bunch period between accelerated bunch and decelerated bunch : $T_b := \min(\mathrm{mod}(T_c, T_a), T_a - \mathrm{mod}(T_c, T_a))$

For a maximum bunch spacing between the charge and discharge, the one's digit of N in 5 is the best chose: $T_b = 6 \times 10^{-9}$ (s)

averaged beam current for accelerated beam: $I_{ma} := 0.1$ (A)

electron charge per bunch (approximate to a point charge): $q := I_{ma} \cdot T_a$ (C) $q = 1.33333333 \times 10^{-9}$ (C)

R/Q normalized induced voltage by a point charge: $V_q(\omega) := \frac{\omega}{2} \cdot q$ (V)

single bunch length 4° of 750MHz: $T_p := \frac{4}{360} \cdot \frac{1}{f_1}$ $T_p = 1.481 \times 10^{-11}$ (s)

bunch spacing between the accelerated/decelerated bunch pairs: $T_{ab} := T_a - T_b$ $T_{ab} = 7.33333333 \times 10^{-9}$ (s)

bunch spacing between the accelerated bunch and decelerated bunch: $T_{bp} := T_b - T_p$ $T_{bp} = 5.98518519 \times 10^{-9}$ (s)

Beam induced voltage at the time just after nth bunch pairs:

$$A(n, Q_{ex}, \omega) := V_q(\omega) \cdot \left(\frac{1 - \exp\left(\frac{-2 \cdot T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot 2 \cdot T_b \right)}{1 - \exp\left(\frac{-T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_b \right)} \right) \cdot \left(\frac{1 - \exp\left(\frac{-n \cdot T_a}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot n \cdot T_a \right)}{1 - \exp\left(\frac{-T_a}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_a \right)} \right)$$

Beam induced voltage during the gap between nth and (n+1)th bunch pairs:

$$B(n, Q_{ex}, \omega, t_1) := A(n, Q_{ex}, \omega) \exp\left(\frac{-t_1}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot t_1 \right)$$

Beam induced voltage at the time just after mth (m=1 or 2) bunch in (n+l)th bunch pairs:

$$C(m,n,Q_{ex},\omega) := B(n,Q_{ex},\omega,T_{ab})\cdot\exp\left[\frac{-m\cdot T_b + T_{bp}}{T_d(Q_{ex},\omega)} + i\cdot\omega\cdot(m\cdot T_b - T_{bp})\right] + V_q(\omega)\cdot\left(\frac{1-\exp\left(\frac{-m\cdot T_b}{T_d(Q_{ex},\omega)} + i\cdot\omega\cdot m\cdot T_b\right)}{1-\exp\left(\frac{-T_b}{T_d(Q_{ex},\omega)} + i\cdot\omega\cdot T_b\right)}\right)$$

Beam induced voltage during the gap between mth and (m+1)th bunches:

$$D(m,n,Q_{ex},\omega,t_2) := C(m,n,Q_{ex},\omega)\cdot\exp\left(\frac{-t_2}{T_d(Q_{ex},\omega)} + i\cdot\omega\cdot t_2\right)$$

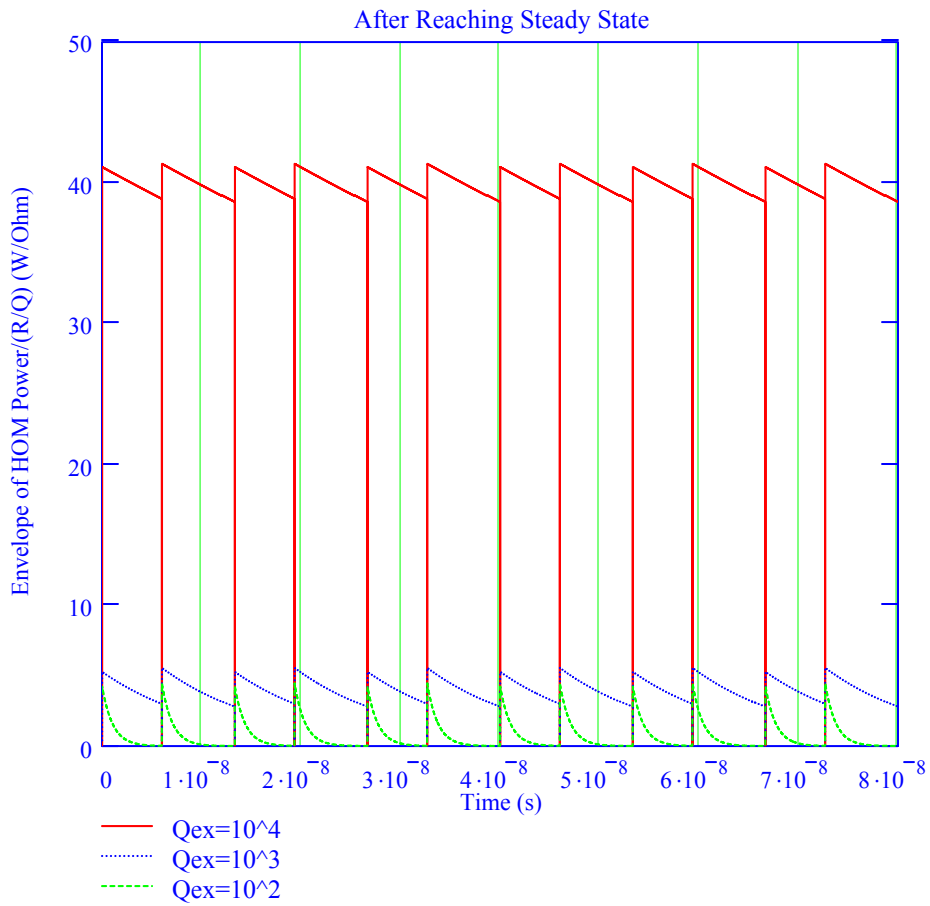
Pulsed beam bunches induced voltage can be expressed as:

$$
V(Q_{ex},\omega,t) := \left|\begin{array}{l}
t_{11} \leftarrow \mathrm{mod}(t, T_a) \\
\text{if } t_{11} \geq 2\cdot T_b \\
\quad \left|\begin{array}{l}
n \leftarrow \mathrm{ceil}\left(\dfrac{t}{T_a}\right) \\
t_1 \leftarrow \mathrm{mod}(t, T_a) - 2\cdot T_b + T_{bp} \\
B(n + 1000, Q_{ex}, \omega, t_1)
\end{array}\right. \\
\text{if } t_{11} < 2\cdot T_b \\
\quad \left|\begin{array}{l}
t_{22} \leftarrow \mathrm{mod}(t_{11}, T_b) \\
\text{if } t_{22} \geq T_p \\
\quad \left|\begin{array}{l}
m \leftarrow \mathrm{ceil}\left(\dfrac{t_{11}}{T_b}\right) \\
t_2 \leftarrow \mathrm{mod}(t_{11}, T_b) - T_p \\
D(m, n + 1000, Q_{ex}, \omega, t_2)
\end{array}\right. \\
V_q(\omega) \quad \text{if } t_{22} < T_p
\end{array}\right.
$$

The HOM power dissipated by the beam is:

$$
P(Q_{ex},\omega,t) := \frac{\left(\left|V(Q_{ex},\omega,t)\right|\right)^2}{Q_{ex}} \qquad t := 0, 1.1\cdot 10^{-12} .. 80\cdot 10^{-9} \quad \text{(s)} \qquad f := 1500\cdot 10^{6} \quad \text{(Hz)} \qquad \omega := 2\cdot\pi\cdot f
$$

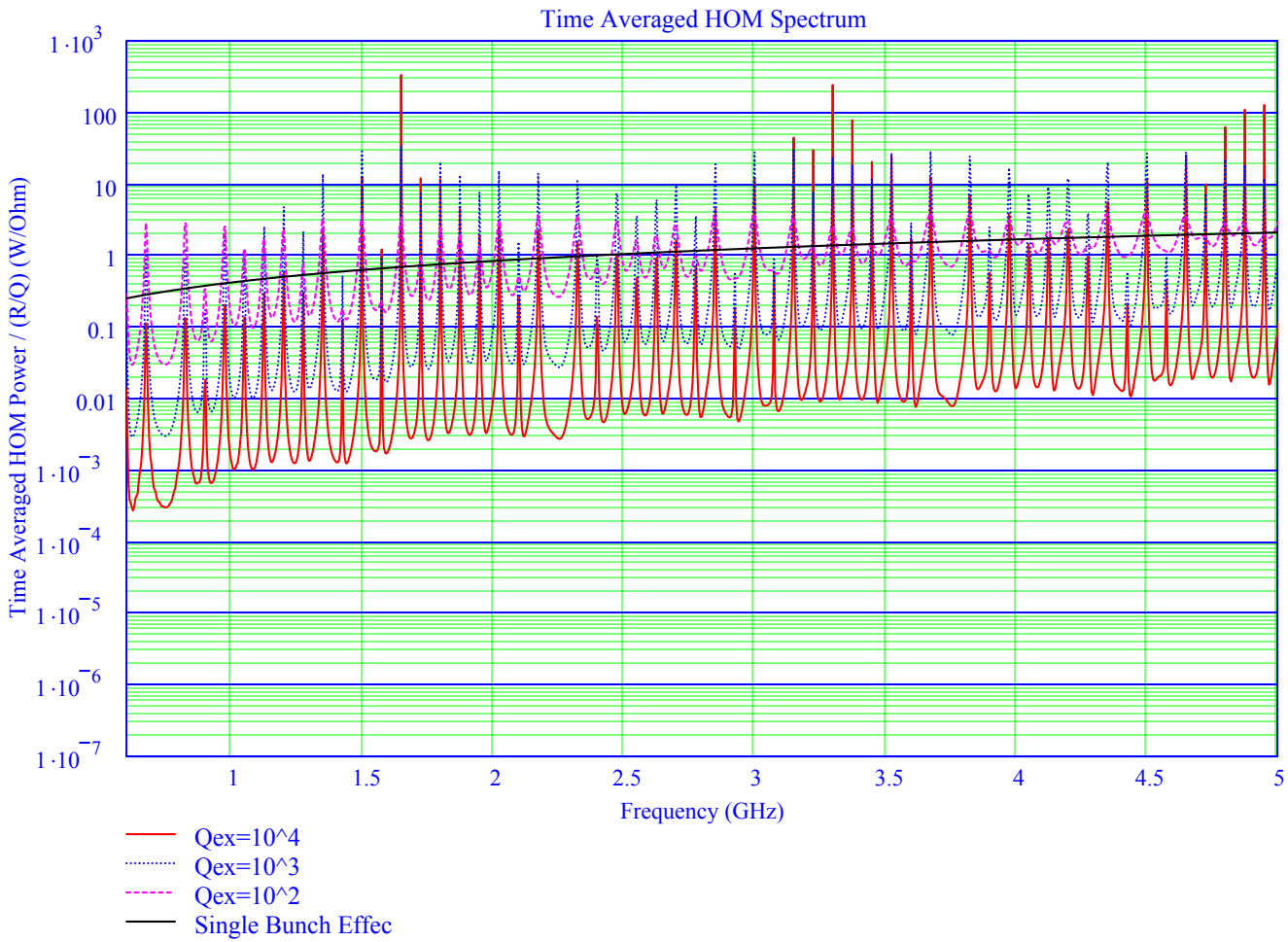
Plot of envelop of HOM power/(R/Q) at f=1.5 GHz (2*750 MHz) after 1000 bunches nearly in steady state:



The time averaged HOM power can be obtained by integrating the $P(Q_{ex},\omega,t)$ in 1000 periods of T_a:

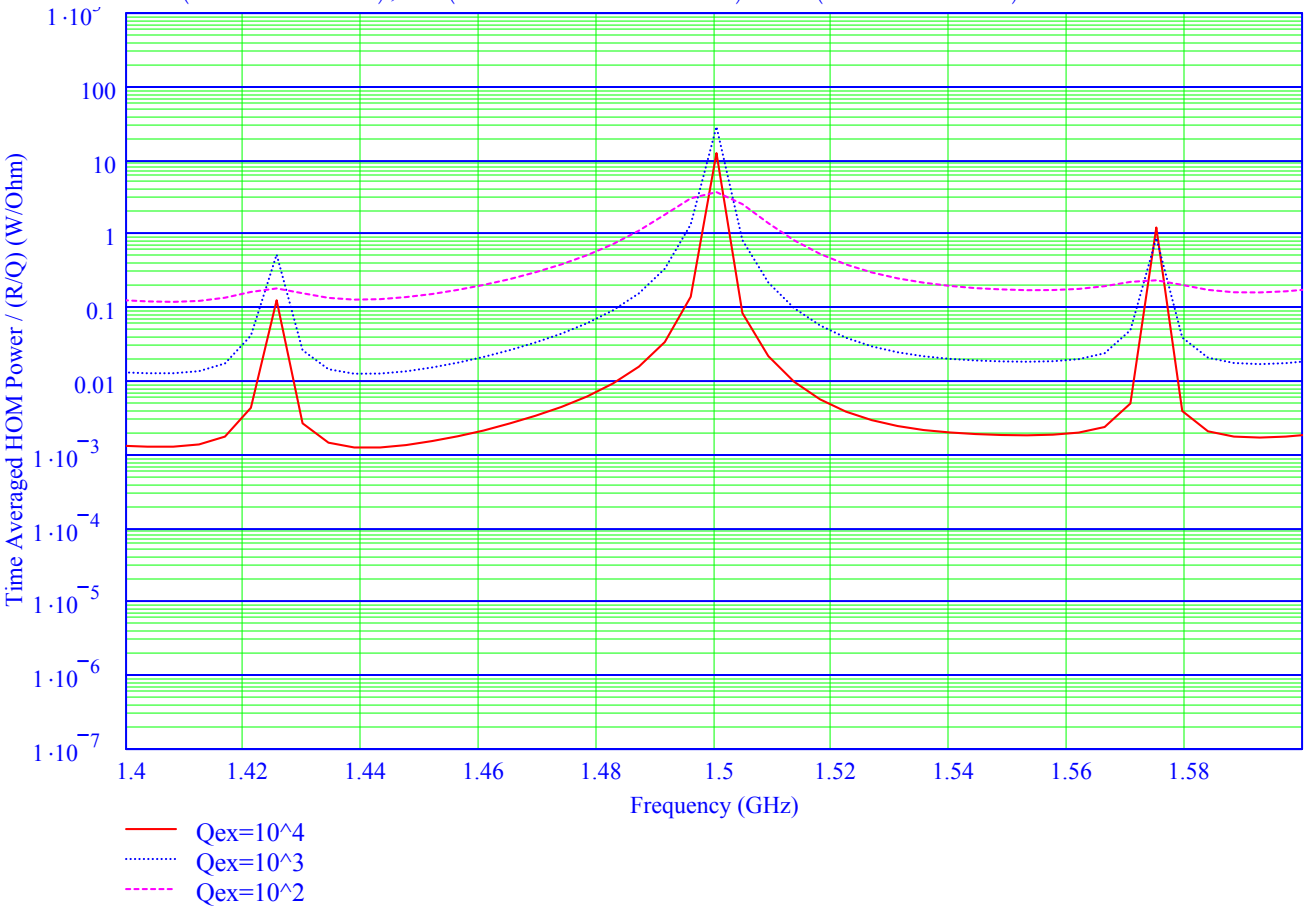
$$P_{ave}(Q_{ex},\omega) := \frac{1}{1000\cdot T_a}\cdot\int_0^{1000\cdot T_a} P(Q_{ex},\omega,t)\,dt$$

$$\omega := 2\cdot\pi\cdot\left(5\cdot10^{9} - 5\cdot10^{9}\right), 2\cdot\pi\cdot\left(5\cdot10^{9} - 5\cdot10^{9} + 4.4\cdot10^{6}\right) .. 2\cdot\pi\cdot\left(5\cdot10^{9} + 5\cdot10^{9}\right)$$



Zoom-in above picture around 1.5GHz peak with finer resolution:

$$\omega := 2\cdot\pi\cdot\left(1.5\cdot10^{9} - 0.1\cdot10^{9}\right), 2\cdot\pi\cdot\left(1.5\cdot10^{9} - 0.1\cdot10^{9} + 2\cdot10^{4}\right) .. 2\cdot\pi\cdot\left(1.5\cdot10^{9} + 0.1\cdot10^{9}\right)$$



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HOM decay time constant: $T_d(Q_{ex}, \omega) := 2 \cdot \frac{Q_{ex}}{\omega}$ (s)

speed of light: $c := 299792458$ (m/s)

SRF cavity's fundamental mode frequency: $f_1 := 750 \cdot 10^6$ (Hz)

photocathode drive laser frequency: $f_2 := 75 \cdot 10^6$ (Hz)

interger number of recirculated path length / fundemantal RF wavelength: $N := 640$

recirculated beam time period: $T_c := \frac{N + \frac{1}{2}}{f_1}$ $T_c = 8.54 \times 10^{-7}$ (s)

bunch period between the accelerated bunches: $T_a := \frac{1}{f_2}$ $T_a = 1.33333333 \times 10^{-8}$ (s)

minor bunch period between accelerated bunch and decelerated bunch : $T_b := \min(\mathrm{mod}(T_c, T_a), T_a - \mathrm{mod}(T_c, T_a))$

For a maximum bunch spacing between the charge and discharge, the one's digit of N in 0 is the worst chose: $T_b = 6.66666667 \times 10^{-10}$

averaged beam current for accelerated beam: $I_{ma} := 0.1$ (A)

electron charge per bunch (approximate to a point charge): $q := I_{ma} \cdot T_a$ (C) $q = 1.33333333 \times 10^{-9}$ (C)

R/Q normalized induced voltage by a point charge: $V_q(\omega) := \frac{\omega}{2} \cdot q$ (V)

single bunch length 4° of 750MHz: $T_p := \frac{4}{360} \cdot \frac{1}{f_1}$ $T_p = 1.481 \times 10^{-11}$ (s)

bunch spacing between the accelerated/decelerated bunch pairs: $T_{ab} := T_a - T_b$ $T_{ab} = 1.26666667 \times 10^{-8}$ (s)

bunch spacing between the accelerated bunch and decelerated bunch: $T_{bp} := T_b - T_p$ $T_{bp} = 6.51851852 \times 10^{-10}$ (s)

Beam induced voltage at the time just after nth bunch pairs:

$$A(n, Q_{ex}, \omega) := V_q(\omega) \cdot \left(\frac{1 - \exp\left(\frac{-2 \cdot T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot 2 \cdot T_b \right)}{1 - \exp\left(\frac{-T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_b \right)} \right) \cdot \left(\frac{1 - \exp\left(\frac{-n \cdot T_a}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot n \cdot T_a \right)}{1 - \exp\left(\frac{-T_a}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_a \right)} \right)$$

Beam induced voltage during the gap between nth and (n+1)th bunch pairs:

$$B(n, Q_{ex}, \omega, t_1) := A(n, Q_{ex}, \omega) \exp\left(\frac{-t_1}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot t_1 \right)$$

Beam induced voltage at the time just after mth (m=1 or 2) bunch in (n+l)th bunch pairs:

$$C(m,n,Q_{ex},\omega) := B(n,Q_{ex},\omega,T_{ab})\cdot\exp\left[\frac{-m\cdot T_b + T_{bp}}{T_d(Q_{ex},\omega)} + i\cdot\omega\cdot(m\cdot T_b - T_{bp})\right] + V_q(\omega)\cdot\left(\frac{1-\exp\left(\frac{-m\cdot T_b}{T_d(Q_{ex},\omega)} + i\cdot\omega\cdot m\cdot T_b\right)}{1-\exp\left(\frac{-T_b}{T_d(Q_{ex},\omega)} + i\cdot\omega\cdot T_b\right)}\right)$$

Beam induced voltage during the gap between mth and (m+1)th bunches:

$$D(m,n,Q_{ex},\omega,t_2) := C(m,n,Q_{ex},\omega)\cdot\exp\left(\frac{-t_2}{T_d(Q_{ex},\omega)} + i\cdot\omega\cdot t_2\right)$$

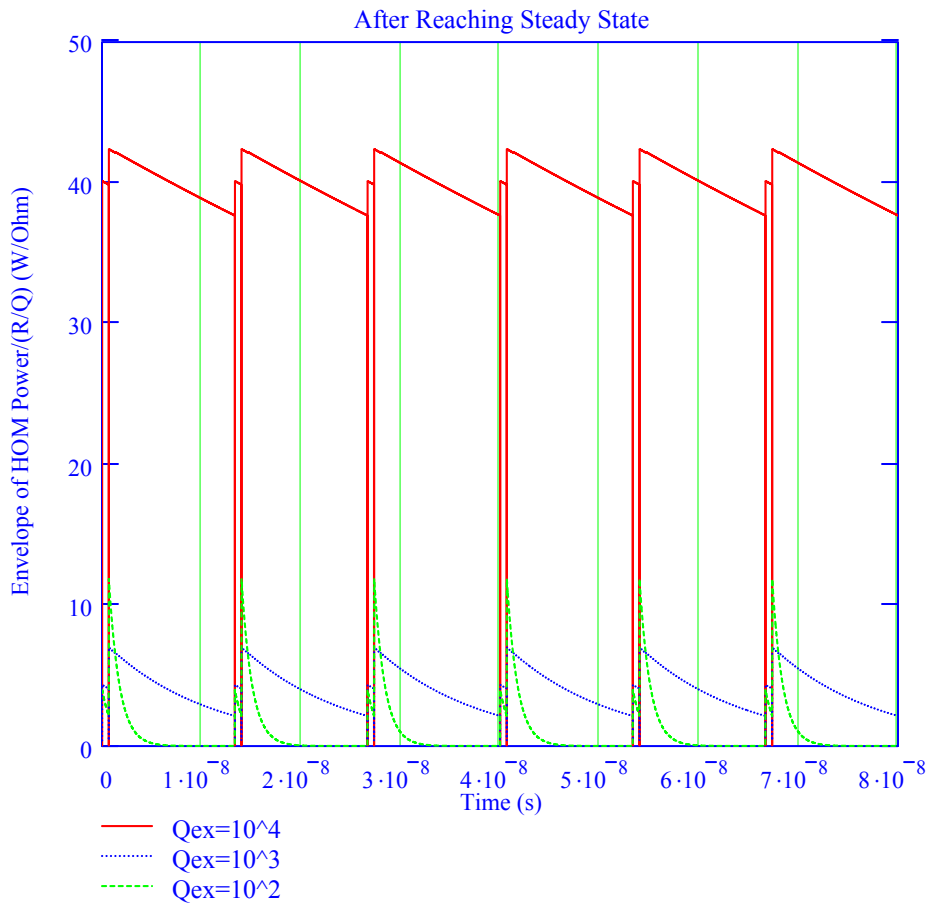
Pulsed beam bunches induced voltage can be expressed as:

$$
V(Q_{ex},\omega,t) := \left|\begin{array}{l}
t_{11} \leftarrow \mathrm{mod}(t, T_a) \\
\text{if } t_{11} \geq 2\cdot T_b \\
\quad \left|\begin{array}{l}
n \leftarrow \mathrm{ceil}\left(\dfrac{t}{T_a}\right) \\
t_1 \leftarrow \mathrm{mod}(t, T_a) - 2\cdot T_b + T_{bp} \\
B(n + 1000, Q_{ex}, \omega, t_1)
\end{array}\right. \\
\text{if } t_{11} < 2\cdot T_b \\
\quad \left|\begin{array}{l}
t_{22} \leftarrow \mathrm{mod}(t_{11}, T_b) \\
\text{if } t_{22} \geq T_p \\
\quad \left|\begin{array}{l}
m \leftarrow \mathrm{ceil}\left(\dfrac{t_{11}}{T_b}\right) \\
t_2 \leftarrow \mathrm{mod}(t_{11}, T_b) - T_p \\
D(m, n + 1000, Q_{ex}, \omega, t_2)
\end{array}\right. \\
V_q(\omega) \quad \text{if } t_{22} < T_p
\end{array}\right.
$$

The HOM power dissipated by the beam is:

$$
P(Q_{ex},\omega,t) := \frac{\left(\left|V(Q_{ex},\omega,t)\right|\right)^2}{Q_{ex}} \qquad t := 0, 1.1\cdot 10^{-12} .. 80\cdot 10^{-9} \quad \text{(s)} \qquad f := 1500\cdot 10^{6} \quad \text{(Hz)} \qquad \omega := 2\cdot\pi\cdot f
$$

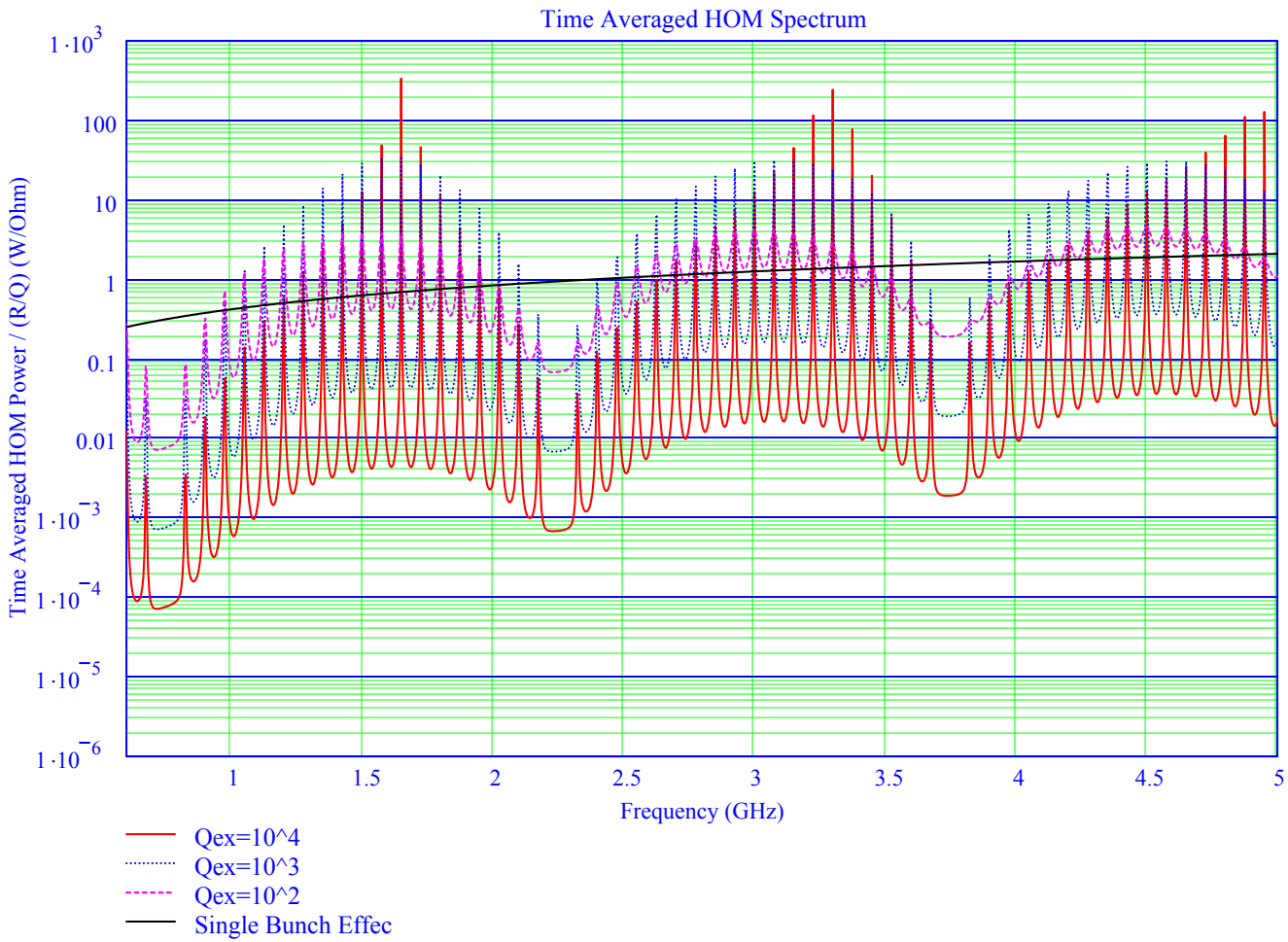
Plot of envelop of HOM power/(R/Q) at f=1.5 GHz (2*750 MHz) after 1000 bunches nearly in steady state:



The time averaged HOM power can be obtained by integrating the $P(Q_{ex},\omega,t)$ in 1000 periods of T_a:

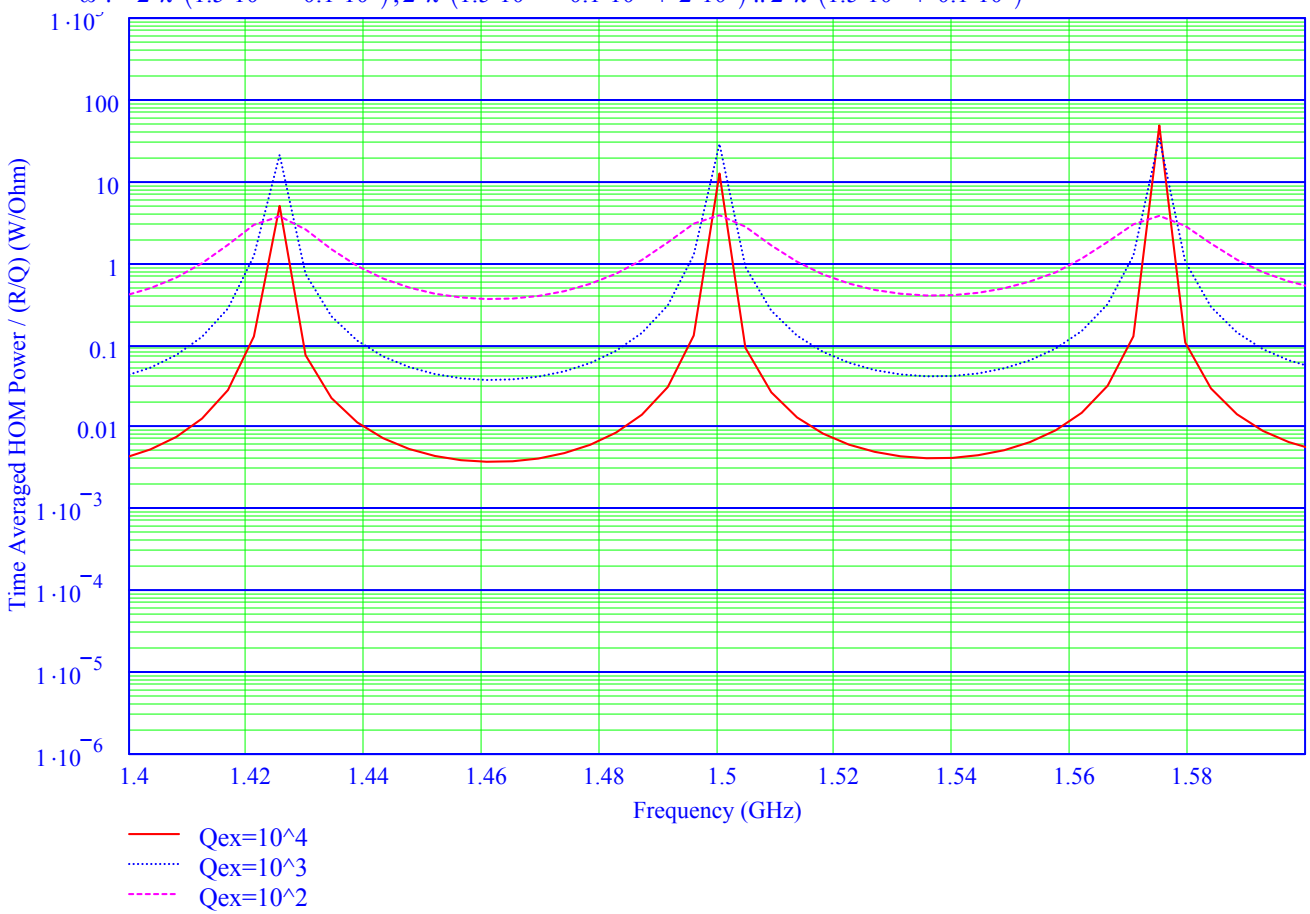
$$P_{ave}(Q_{ex},\omega) := \frac{1}{1000\cdot T_a}\cdot\int_0^{1000\cdot T_a} P(Q_{ex},\omega,t)\,dt$$

$$\omega := 2\cdot\pi\cdot\left(5\cdot10^{9} - 5\cdot10^{9}\right), 2\cdot\pi\cdot\left(5\cdot10^{9} - 5\cdot10^{9} + 4.4\cdot10^{6}\right) .. 2\cdot\pi\cdot\left(5\cdot10^{9} + 5\cdot10^{9}\right)$$



Zoom-in above picture around 1.5GHz peak with finer resolution:

$$\omega := 2\cdot\pi\cdot\left(1.5\cdot10^{9} - 0.1\cdot10^{9}\right), 2\cdot\pi\cdot\left(1.5\cdot10^{9} - 0.1\cdot10^{9} + 2\cdot10^{4}\right) .. 2\cdot\pi\cdot\left(1.5\cdot10^{9} + 0.1\cdot10^{9}\right)$$



Time Structure of JLab FEL Upgrade (100mA, Pulsed) Tune-up Beam & HOM Induced Voltage Analysis Based on Sangho Kim's SNS/AP Technical Note No.10 and Correction from His Mathematica File

Haipeng Wang, Last updated on April 28, 2005

HOM decay time constant: $T_d(Q_{ex}, \omega) := 2 \cdot \frac{Q_{ex}}{\omega}$ (s)

speed of light: $c := 299792458$ (m/s)

SRF cavity's fundamental mode frequency: $f_1 := 750 \cdot 10^6$ (Hz)

photocathode drive laser frequency: $f_2 := 75 \cdot 10^6$ (Hz)

interger number of recirculated path length / fundemantal RF wavelength: $N := 645$

recirculated beam time period: $T_c := \frac{N + \frac{1}{2}}{f_1}$ $T_c = 8.607 \times 10^{-7}$ (s)

bunch period between the accelerated bunches: $T_a := \frac{1}{f_2}$ $T_a = 1.33333333 \times 10^{-8}$ (s)

minor bunch period between accelerated bunch and decelerated bunch : $T_b := \min(\mathrm{mod}(T_c, T_a), T_a - \mathrm{mod}(T_c, T_a))$

For a maximum bunch spacing, the one's digit of N in 4 or 5 is the best chose: $T_b = 6 \times 10^{-9}$ (s)

single bunch length 4° of 750MHz: $T_p := \frac{4}{360} \cdot \frac{1}{f_1}$ $T_p = 1.481 \times 10^{-11}$ (s)

bunch spacing between the accelerated/decelerated bunch pairs: $T_{ab} := T_a - T_b$ $T_{ab} = 7.33333333 \times 10^{-9}$ (s)

bunch spacing between the accelerated bunch and decelerated bunch: $T_{bp} := T_b - T_p$ $T_{bp} = 5.98518519 \times 10^{-9}$ (s)

averaged macro-pulse beam current for accelerated beam: $I_{ma} := 0.1$ (A)

electron charge per bunch (approximate to a point charge): $q := I_{ma} \cdot T_a$ $q = 1.33333333 \times 10^{-9}$ (C)

Pulsed mode macro-pulse repeatation rate: $f_m := 60$ (Hz)

macro-pulse period: $T_m := \frac{1}{f_m}$ $T_m = 0.01666667$ (s)

macro-pulse length: $T_{mb} := 250 \cdot 10^{-6}$ (s)

The gap between each macro-pulses: $T_G := T_m - T_{mb}$ $T_G = 0.01641667$ (s)

peak macro-pulse beam current for accelerated beam: $I_{mp} := I_{ma} \cdot \frac{T_m}{T_{mb}}$ $I_{mp} = 6.66666667$ (A)

Total number of accelerated/decelerated bunch pairs within one macro-pulse: $N := \mathrm{trunc}\left(\frac{T_{mb}}{T_a}\right)$ $N = 18750$

R/Q normalized induced voltage by a point charge: $V_q(\omega) := \frac{\omega}{2} \cdot q$ (V)

Beam induced voltage at the time just after kth macro-pulse:

$$A(k, Q_{ex}, \omega) := V_q(\omega) \cdot \left(\frac{1 - \exp\left(\frac{-2 \cdot T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot 2 \cdot T_b \right)}{1 - \exp\left(\frac{-T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_b \right)} \right) \cdot \left(\frac{1 - \exp\left(\frac{-N \cdot T_a}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot N \cdot T_a \right)}{1 - \exp\left(\frac{-T_a}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_a \right)} \right) \cdot \left(\frac{1 - \exp\left(\frac{-k \cdot T_m}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot k \cdot T_m \right)}{1 - \exp\left(\frac{-T_m}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_m \right)} \right)$$

Beam induced voltage during the gap between kth and (k+1)th macro-pulses:

$$B(k, Q_{ex}, \omega, t_1) := A(k, Q_{ex}, \omega) \exp\left(\frac{-t_1}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot t_1 \right)$$

Beam induced voltage at the time just after nth bunch pair in (k+1)th macro-pulse:

$$C(n, k, Q_{ex}, \omega) := B(k, Q_{ex}, \omega, T_G) \cdot \exp\left[\frac{-n \cdot T_a + T_{ab}}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot (n \cdot T_a - T_{ab}) \right] + V_q(\omega) \cdot \left(\frac{1 - \exp\left(\frac{-2 \cdot T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot 2 \cdot T_b \right)}{1 - \exp\left(\frac{-T_b}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_b \right)} \right) \cdot \left(\frac{1 - \exp\left(\frac{-n \cdot T_a}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot n \cdot T_a \right)}{1 - \exp\left(\frac{-T_a}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot T_a \right)} \right)$$

Beam induced voltage during the gap between nth and (n+1) bunch pairs in (k+l)th macro-pulse:

$$D(n, k, Q_{ex}, \omega, t_2) := C(n, k, Q_{ex}, \omega) \cdot \exp\left(\frac{-t_2}{T_d(Q_{ex}, \omega)} + i \cdot \omega \cdot t_2 \right)$$

Beam induced voltage at the time just after mth (m=1 or 2) bunch in (n+1)th bunch pair and in (k+1)th macro-pulse:

$$E(m,n,k,Q_{ex},\omega) := D(n,k,Q_{ex},\omega,T_{ab})\cdot\exp\left[\frac{-m\cdot T_b + T_{bp}}{T_d(Q_{ex},\omega)} + i\cdot\omega\cdot(m\cdot T_b - T_{bp})\right] + V_q(\omega)\cdot\left(\frac{1-\exp\left(\frac{-m\cdot T_b}{T_d(Q_{ex},\omega)} + i\cdot\omega\cdot m\cdot T_b\right)}{1-\exp\left(\frac{-T_b}{T_d(Q_{ex},\omega)} + i\cdot\omega\cdot T_b\right)}\right)$$

Beam induced voltage during the gap between mth and (m+1)th bunches in (n+1)th bunch pair and in (k+1)th macro-pulse:

$$F(m,n,k,Q_{ex},\omega,t_3) := E(m,n,k,Q_{ex},\omega)\cdot\exp\left(\frac{-t_3}{T_d(Q_{ex},\omega)} + i\cdot\omega\cdot t_3\right)$$

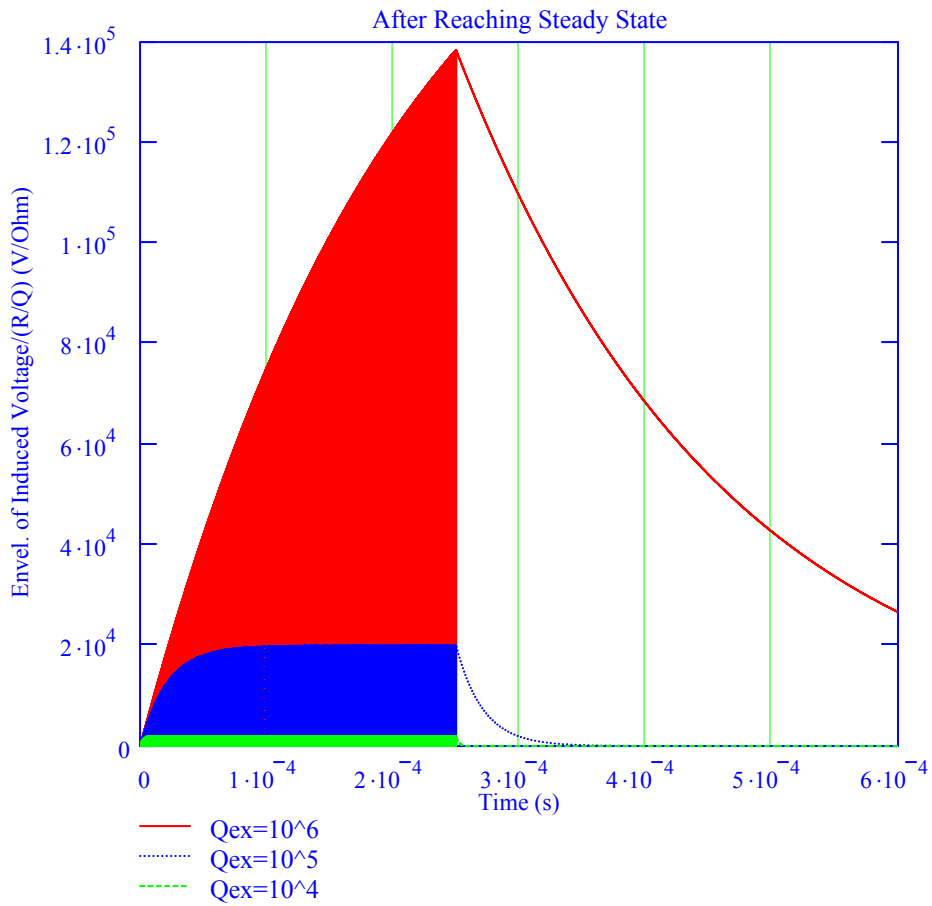
Pulsed beam bunches induced voltage can be expressed as:

$$
\begin{aligned}
V(Q_{ex},\omega,t) := \;& t_{11} \leftarrow \mathrm{mod}(t, T_m) \\
& \text{if } t_{11} \geq T_{mb} \\
& \quad k \leftarrow \mathrm{ceil}\left(\frac{t}{T_m}\right) \\
& \quad t_1 \leftarrow \mathrm{mod}(t, T_m) - T_{mb} \\
& \quad B(k + 1000, Q_{ex}, \omega, t_1) \\
& \text{if } t_{11} < T_{mb} \\
& \quad t_{22} \leftarrow \mathrm{mod}(t_{11}, T_a) \\
& \quad \text{if } t_{22} \geq 2 \cdot T_b \\
& \quad\quad n \leftarrow \mathrm{ceil}\left(\frac{t_{11}}{T_a}\right) \\
& \quad\quad t_2 \leftarrow \mathrm{mod}(t_{11}, T_a) - 2 \cdot T_b + T_{bp} \\
& \quad\quad D(n, k + 1000, Q_{ex}, \omega, t_2) \\
& \quad \text{if } t_{22} < 2 \cdot T_b \\
& \quad\quad t_{33} \leftarrow \mathrm{mod}(t_{22}, T_b) \\
& \quad\quad \text{if } t_{33} \geq T_p \\
& \quad\quad\quad m \leftarrow \mathrm{ceil}\left(\frac{t_{22}}{T_b}\right) \\
& \quad\quad\quad t_3 \leftarrow \mathrm{mod}(t_{22}, T_b) - T_p \\
& \quad\quad\quad F(m, n, k + 1000, Q_{ex}, \omega, t_3) \\
& \quad\quad V_q(\omega) \;\; \text{if } t_{33} < T_p
\end{aligned}
$$

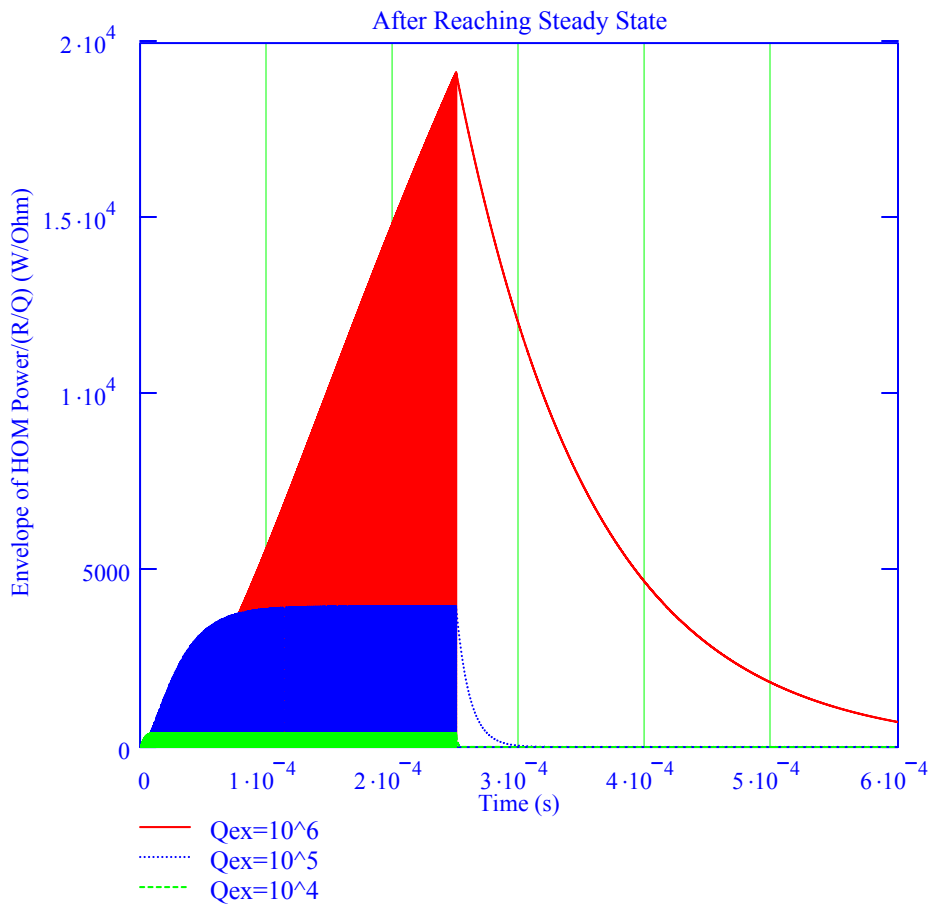
The HOM power dissipated by the beam is:

$$P(Q_{ex},\omega,t) := \frac{\left(\left|V(Q_{ex},\omega,t)\right|\right)^2}{Q_{ex}} \qquad f := 1500\cdot 10^6 \quad \text{(Hz)} \qquad \omega := 2\cdot\pi\cdot f \qquad t := 0, 1.1\cdot 10^{-8} .. 60\cdot 10^{-5} \quad \text{(s)}$$

Plot of envelop of the induced voltage/(R/Q) at f=1.5 GHz (2*750 MHz) after 1000 macro-pulses in steady state:



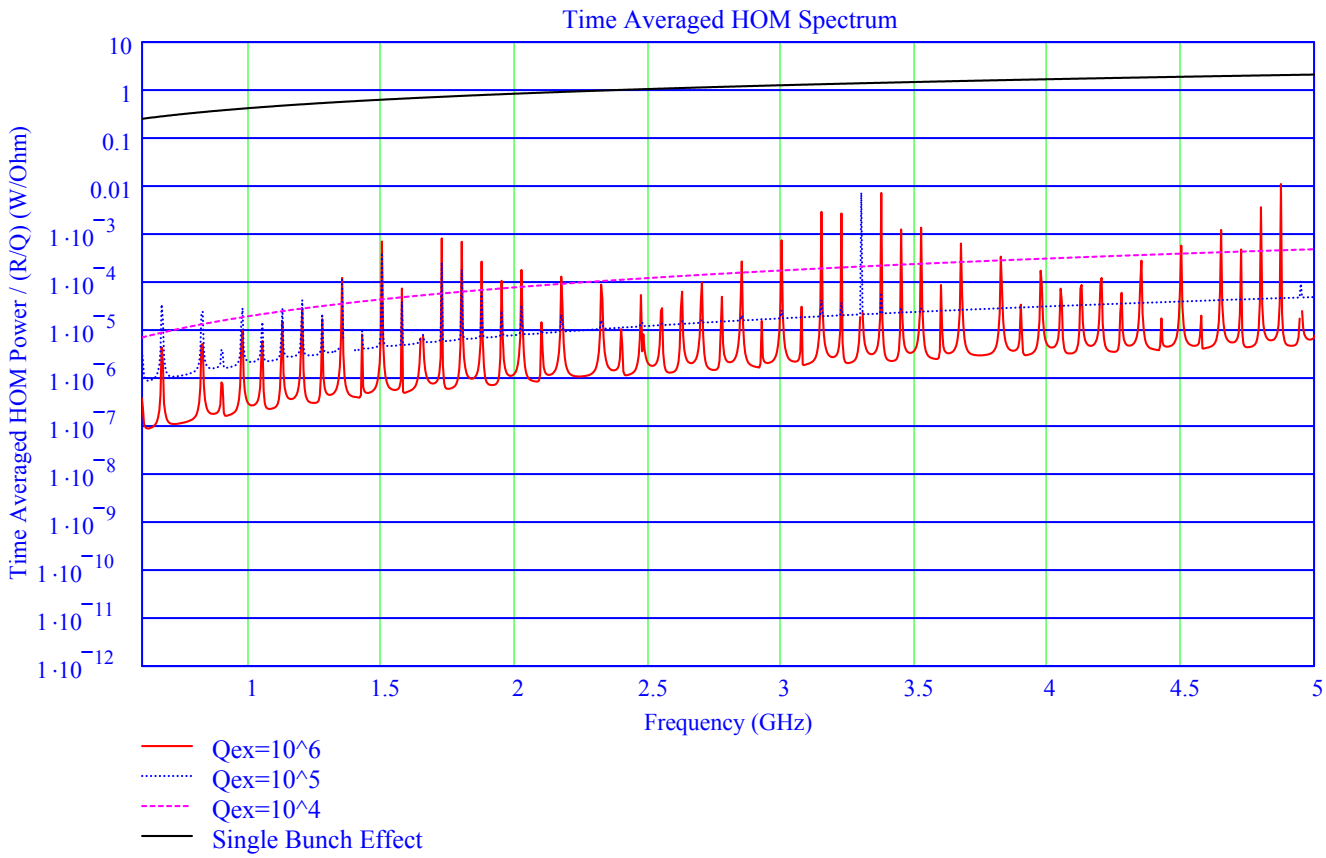
Plot of envelop of HOM power/(R/Q) at f=1.5 GHz (2*750 MHz) after 1000 bunches nearly in steady state:

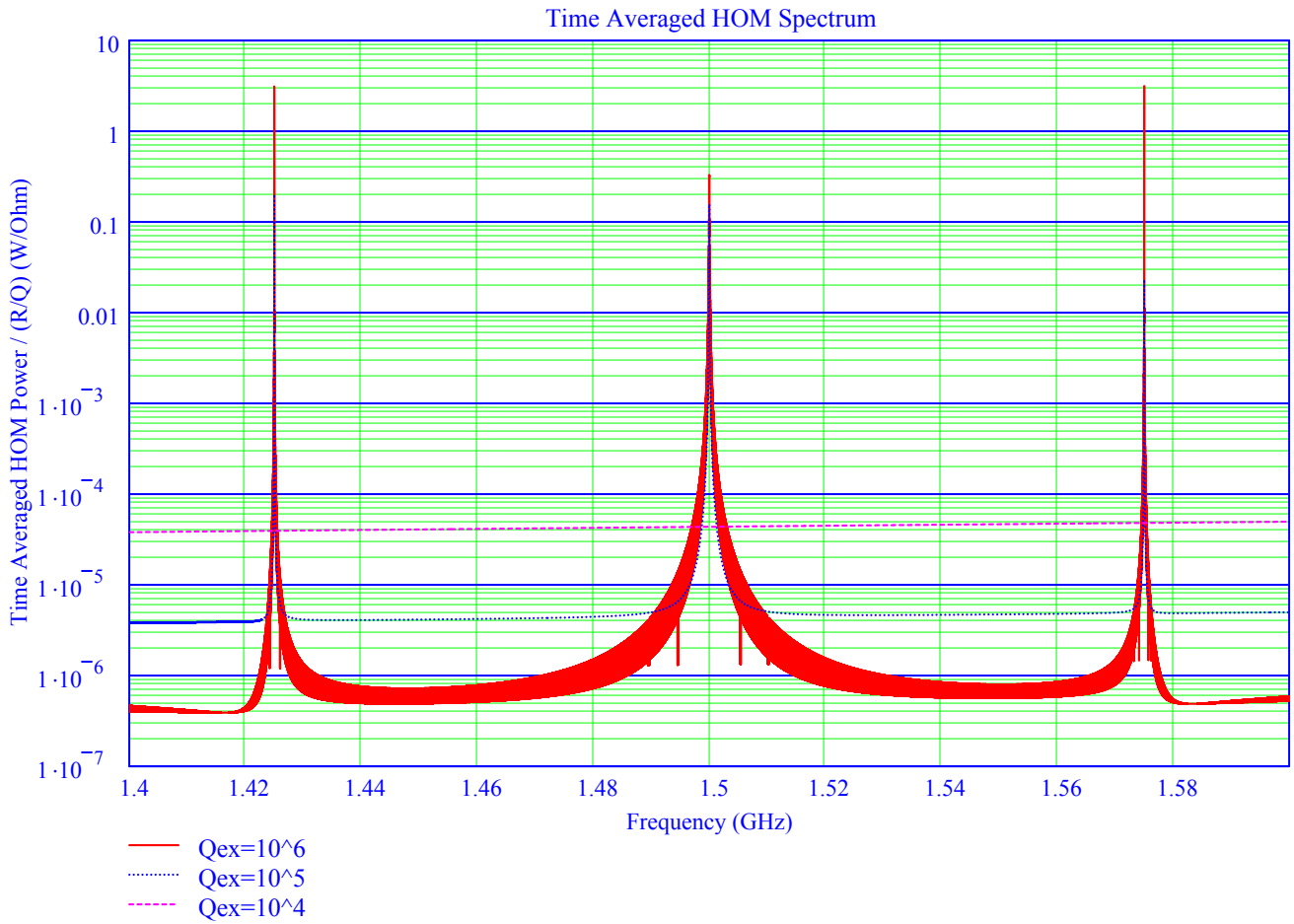


The time averaged HOM power can be obtained by integrating the $P(Q_{ex},\omega,t)$ in 1 periods of T_m:

$$P_{ave}(Q_{ex},\omega) := \frac{1}{T_m}\cdot\int_0^{T_m} P(Q_{ex},\omega,t)\,dt$$

$$\omega := 2\cdot\pi\cdot\left(5\cdot10^9 - 5\cdot10^9\right), 2\cdot\pi\cdot\left(5\cdot10^9 - 5\cdot10^9 + 4.4\cdot10^6\right) .. 2\cdot\pi\cdot\left(5\cdot10^9 + 5\cdot10^9\right)$$



Zoom-in above picture around 1.5GHz peak with finer resolution:

$$\omega := 2\cdot\pi\cdot\left(1.5\cdot10^{9} - 0.1\cdot10^{9}\right), 2\cdot\pi\cdot\left(1.5\cdot10^{9} - 0.1\cdot10^{9} + 1\cdot10^{4}\right) .. 2\cdot\pi\cdot\left(1.5\cdot10^{9} + 0.1\cdot10^{9}\right)$$