

Simulations of Cumulative Beam Breakup in the AES injector

Eduard Pozdeyev

Abstract

In this paper, I simulate cumulative (single-pass) beam breakup (BBU). The results of these simulations are used to determine required damping of dipole high order modes (HOM).

Acceleration and RF focusing in the injector

There are three identical single-cell accelerating cavities in the injector. Each cavity operates at 748.5 MHz. The quality factor and R/Q of dangerous dipole HOMs were obtained in RF simulations described elsewhere and provided to me by Genfa Wu. The frequency of the HOM was measured at AES using a copper model and was provided to me by Genfa too. Table 1 shows the provided HOM parameters. I'll refer to this data as "nominal set of HOM parameters". The accelerating field profile is shown in Figure 1.

Mode frequency	Q	R/Q	Polarization
1040.685	8.59e3	2.2	X
1041.233	5.54e3	2.2	Y
1044.813	2.44e4	9.5	X
1045.030	3.04e4	9.5	Y

In the simulations, the length of each cavity was set to 56.57 cm, which was the length of the field profile shown in Figure 1. The distance between cavity mid-points was 71.97 cm. The initial kinetic beam energy was 500 keV. The phase of each cavity was adjusted for the maximum energy gain while the field magnitude was adjusted simultaneously in all the three cavities to obtain a final kinetic energy of 7.5 MeV.

An electron traveling through a cavity off-axis is focused by the RF electric and magnetic fields. To include the cavity focusing, I have written a short C++ code that calculates trajectories of two particles with initial conditions given by the vectors $(a, 0)$ and $(0, b)$, where a is of the order of 1 mm and b is of the order of $1.0e-3$. To calculate the trajectories, the code solves the system of the equations of motion using the classical 4th order Runger-Kutta method. The electro-magnetic field is given by

$$\begin{aligned}E_z &= E_{z0} \cos(\omega t + \varphi_0) \\E_r &= -\frac{r}{2} \frac{\partial E_{z0}}{\partial z} \cos(\omega t + \varphi_0) \\B_\theta &= -\frac{r}{2} k E_{z0} \sin(\omega t + \varphi_0)\end{aligned}$$

where E_{z0} is the accelerating field on the cavity axis. ω and k are the angular frequency and the wave number of the accelerating field respectively. After the code calculates the trajectories, it calculates the matrix of the transformation as $m_{11}=x_1/a$, $m_{21}=x'_1/a$,

$m_{12}=x_2/b$, $m_{22}=x'_2/b$, where (x_1, x'_1) and (x_2, x'_2) are phase space vectors at the exit of the cavity corresponding to $(a, 0)$ and $(0, b)$ respectively. This procedure was used to calculate transfer matrices for each cavity separately because the phase corresponding to maximum energy gain is different for each cavity. Actually, matrices were calculated for each half of each cavity for reasons that will be explained later.

Simulation procedure

A modified version of my BBU code, which is temporarily called ERLBBU, was used for these simulations. The code assumes impulse approximation, treating HOMs as infinitely short kicks. The HOM voltage has two components: real and imaginary. The real component of the field is proportional to the electric HOM field. The imaginary component is proportional to the magnetic HOM field. A bunch passing through a cavity modifies the real component of the HOM field according to

$$\delta V_r = q \frac{c}{2} \left(\frac{\omega_{HOM}}{c} \right)^2 \left(\frac{R}{Q} \right)_{HOM} (x \cdot \cos(\alpha) + y \cdot \sin(\alpha)),$$

where α is HOM polarization angle respective to the x-y frame. The deflection angle produced by the imaginary part of the HOM voltage is given by

$$\delta x' = \frac{V_i \cos(\alpha)}{V_b}, \quad \delta y' = \frac{V_i \sin(\alpha)}{V_b},$$

where V_b is the beam voltage, pc/q . The HOM voltage is updated between bunch passages according to

$$\begin{bmatrix} V_r \\ V_i \end{bmatrix}_{t+\delta t} = \exp\left(-\frac{\omega_{HOM} \delta t}{2Q_{HOM}}\right) \begin{bmatrix} \cos(\omega_{HOM} \delta t) & -\sin(\omega_{HOM} \delta t) \\ \sin(\omega_{HOM} \delta t) & \cos(\omega_{HOM} \delta t) \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix}_t$$

To propagate particles between HOMs the code uses 4x4 transfer matrices. In these simulations, each cavity was split into two equal halves. The matrix of each half was calculated according to the procedure described above. All the HOMs were situated in the middle of each cavity.

The beam was offset from the equilibrium trajectory by 5 mm in both x- and y-directions at the entrance to the first cavity. Figure 2 shows the horizontal projection of the beam trajectory without the effect of HOMs. The trajectory is shown for an initial displacement of 1 mm.

The beam dynamics was simulated for two different bunch repetition frequencies: 74.85 MHz and 748.5 MHz. In both cases, the beam current was the same, 100 mA, that means higher charge per bunch in the case of 74.85 MHz.

The HOM data provided by Genfa Wu and shown in Table 1 describes a set of HOM parameters for a single cavity. To simulate the spread of the HOM frequency three

cases were simulated: 1) the frequency of corresponding HOMs was the same in all the three cavities and equal to the nominal values (see Table 1), 2) the frequency of HOMs in the first and the second cavities was equal to the nominal values while the frequency of each HOM in the third cavity was higher than the frequency of HOMs in the first two cavities by 10 MHz, 3) the frequency of HOMs in the first cavity was equal to the nominal values, the frequency of HOMs in the second cavity was lower than the nominal values by 10 MHz, the frequency of HOMs in the third cavity was higher than nominal values by 10 MHz.

Additionally, for each case described in the previous paragraph, the frequency of HOMs was varied by ± 5 MHz. The frequency of all the HOMs was changed simultaneously by the same amount. The frequency variation was performed by another C++ code that read initial parameters as the input, generated input files for ERLBBU, run ERLBBU, waited until ERLBBU finishes, and then repeated the whole procedure for another set of frequencies.

The required simulation time increases with the quality factor of simulated HOMs. For a Q of $1e9$, the beam dynamics in the injector was simulated for 15 seconds of the “injector time”. For a bunch repetition frequency of 748.5 MHz, the actual time required to accomplish a single run for a fixed set of HOM frequencies was several hours. To keep the total simulation time within reasonable limits, I have decreased the number of frequency steps for high- Q cases. The number of steps was chosen as follows: 5000 for $Q=1e4$ and $Q=1e5$, 500 for $Q=1e6$, 50 for $Q=1e7$, 5 for $Q=1e8$, and 1 for $Q=1e9$.

An example of cumulative BBU in the AES injector for a fixed HOM frequency is shown in Figure 3. The figure shows the x - and y - displacements after the third cavity. In the figure, two regimes can be clearly distinguished: the transition and the steady state. In the transition regime, the amplitude of oscillations grows and then decays. The beam oscillates around the steady state displacement. After the transition oscillations die out, the beam displacement stays constant. The magnitude of the steady state displacement depends on the charge per bunch. The code changing the HOM frequency finds out the maximum amplitude of oscillations during the transition from the output generated by ERLBBU and records it as a function of the HOM frequency shift.

To determine the required damping of HOMs the quality factor of HOMs was changed from its nominal value given in Table 1. The quality factor of HOMs was incremented by 5 orders of magnitude in steps of one order of magnitude. The quality factor of all the HOMs in all the cavities was incremented simultaneously. The mantissa of the quality factor was unchanged. For simplicity, I'll refer to the initial set of Q 's as $Q=1e4$. All the other cases I'll refer as $Q= 1e5, 1e6\dots 1e9$.

The maximum allowed transition amplitude of coordinate and angular oscillations was set to $5e-4$ m or rad. respectively. If the amplitude of oscillations during a frequency scan for a given Q exceeded $5e-4$, a better damping of HOM was required.

Results for $f_b=748.5$ MHz

a. Frequency of corresponding HOMs is the same in all the three cavities. Figure 4 shows the maximum amplitude of oscillations during the transition period as a function of frequency difference from the nominal values. According to the figure, it is sufficient

to damp the quality factor of HOMs to $1e6$ to satisfy the requirement outlined in the previous section.

b. Frequency of corresponding HOMs is the same in the first two cavities. Frequency of HOMs in the third cavity is higher by 10 MHz. Figure 5 shows the maximum amplitude of oscillations during the transition period as a function of frequency difference from the initial numbers. According to the figure, it is sufficient to damp the quality factor of HOMs to $1e8$ to satisfy the requirement outlined in the previous section.

c. Frequency of HOMs is different in all the three cavities. Frequency of HOMs in the second cavity is lower than the frequency of HOMs in the first cavity by 10 MHz. Frequency of HOMs in the third cavity is higher than the frequency of HOMs in the first cavity by 10 MHz. At the beginning of the frequency scan, the frequency of HOMs in the first cavity is equal to the nominal values (Table 1). Figure 6 shows the maximum amplitude of oscillations during the transition period as a function of the frequency difference. According to the figure, no HOM damping is required to satisfy the requirement outlined in the previous section.

Results for $f_b=74.85$ MHz

a. Frequency of corresponding HOMs is the same in all the three cavities. Figure 7 shows the maximum amplitude of oscillations during the transition period as a function of the frequency difference from the nominal numbers. Because of the resonance between HOMs and the 14th harmonic of the bunch repetition frequency the amplitude of oscillations exceeds $5e-4$ even if Q is $1e4$. However, if Q is $1e5$ or lower, the range of frequencies where the amplitude of oscillations can reach dangerous magnitudes is small.

b. Frequency of corresponding HOMs is the same in the first two cavities. Frequency of HOMs in the third cavity is higher by 10 MHz. Figure 8 shows the maximum amplitude of oscillations during the transition period as a function of the frequency difference from the initial numbers. Because of the resonance between HOMs and the 14th harmonic of the bunch repetition frequency the amplitude of oscillations exceeds $5e-4$ even if $Q=1e5$. However, if Q is $1e6$ or lower, the range of frequencies where the amplitude of oscillations can reach dangerous magnitudes is small.

c. Frequency of HOMs is different in all the three cavities. Frequency of HOMs in the second cavity is smaller than the frequency of HOMs in the first cavity by 10 MHz. Frequency of HOMs in the third cavity is higher than the frequency of HOMs in the first cavity by 10 MHz. At the beginning of the frequency scan, the frequency of HOMs in the first cavity is equal to the nominal values (Table 1). Figure 9 shows the maximum amplitude of oscillations during the transition period as a function of frequency difference from the nominal numbers. Because of the resonance between HOMs and the 14th harmonic of the bunch repetition frequency the amplitude of oscillations exceeds $5e-4$ even if $Q=1e5$. However, if Q is $1e6$ or lower, the range of frequencies where the amplitude of oscillations can reach dangerous magnitudes is small.

Discussion and Conclusions

Because precise information on the frequency of HOM's is not available, it is impossible to accurately estimate the required HOM damping to obtain sufficiently small oscillations during the transition period. However, a situation when the frequency of corresponding HOMs in all the three cavities is the same is highly unlikely. Although, a coincidence of the frequency of corresponding HOMs in two cavities is also unlikely for high Q 's, I assume that this is the case and derive requirements on HOM damping assuming that the frequency of corresponding HOMs is the same in the first two cavities. Under this assumption, I conclude from the simulations that the quality factor of HOMs has to be damped to approx. $1e8$ in the case of the bunch repetition rate equal to 748.5 MHz. For a bunch repetition rate of 74.85 MHz, the resonance between the 14th harmonic of the bunch repetition rate and the HOMs can cause the oscillations to grow to unacceptably large amplitudes even if the quality factor of HOMs is $1e5$. However, if Q is $1e6$ or lower, the range of frequencies where the amplitude of oscillations can reach dangerous magnitudes becomes small.

If the frequency of corresponding HOMs is the same in all the cavities, the HOMs have to be damped to $1e6$ and $1e5$ for a bunch repetition rate of 748.5 and 74.85 MHz respectively. If the frequency of corresponding HOMs is different in all the three cavities the HOMs have to be damped to $1e6$ for a bunch repetition rate of 74.85 MHz. No damping is required in this case if the bunch repetition frequency is 748.5 MHz.

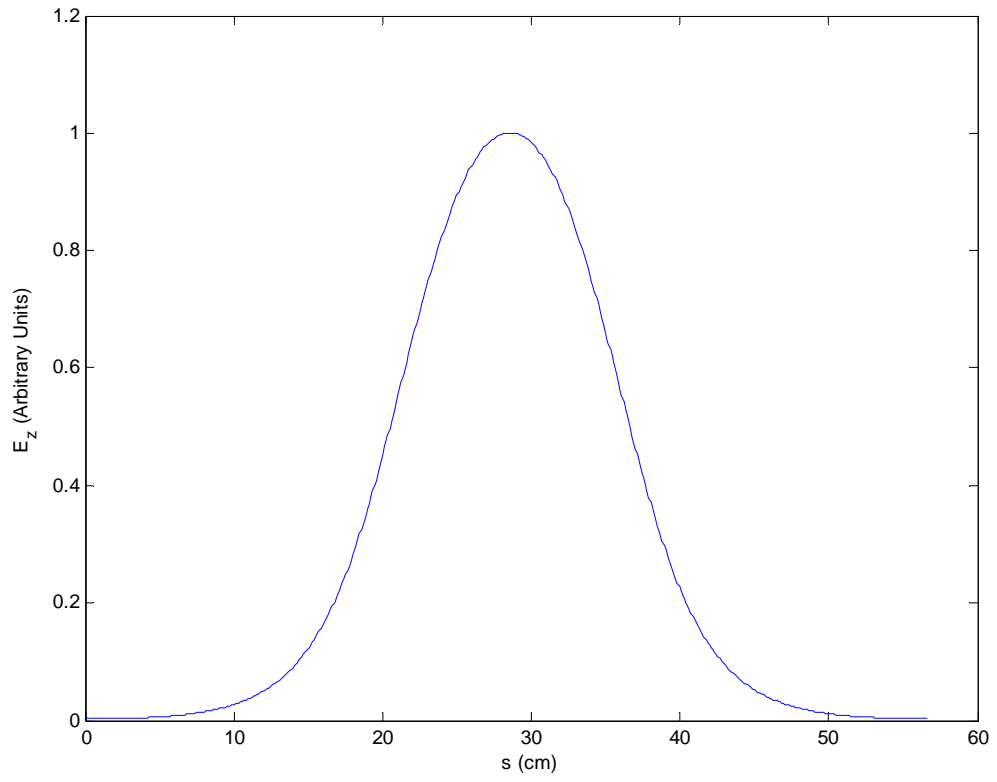


Figure 1: Accelerating field profile.

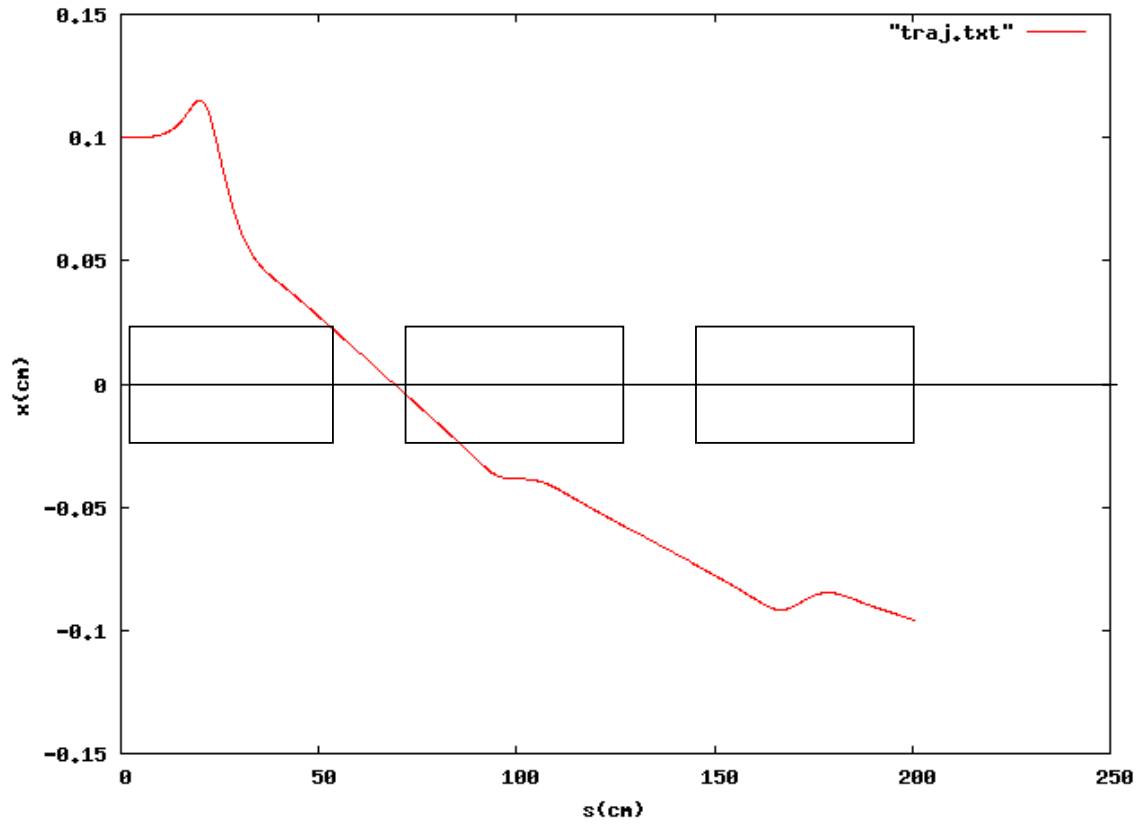


Figure 2: A particle trajectory with an initial displacement of 1 mm. The particle trajectory is deflected by the cavity RF fields. The effect of HOMs is not included. Black empty rectangles represent accelerating cavities.

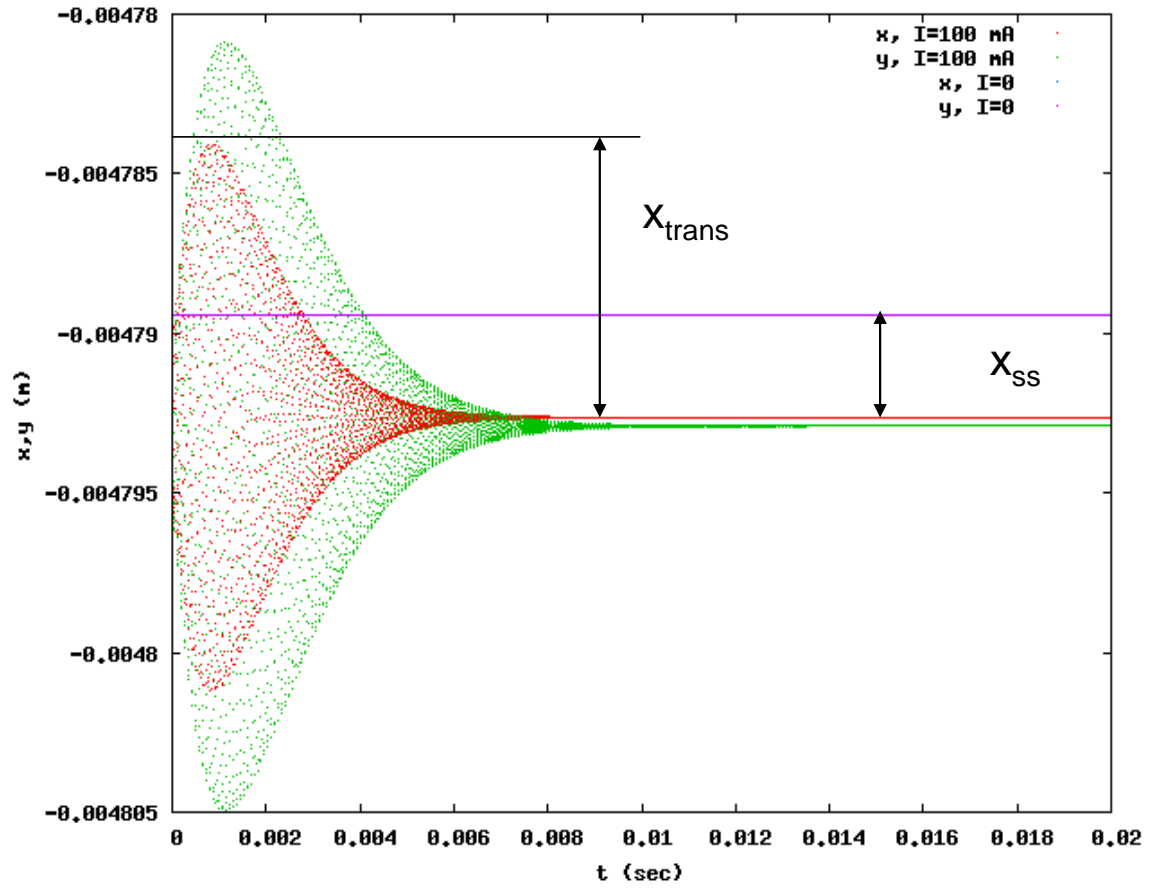


Figure 3: Particle vertical and horizontal displacements at the end of the third cavity. X_{trans} is the maximum amplitude of oscillations during the transition. X_{ss} is the steady state displacement from the zero-current trajectory.

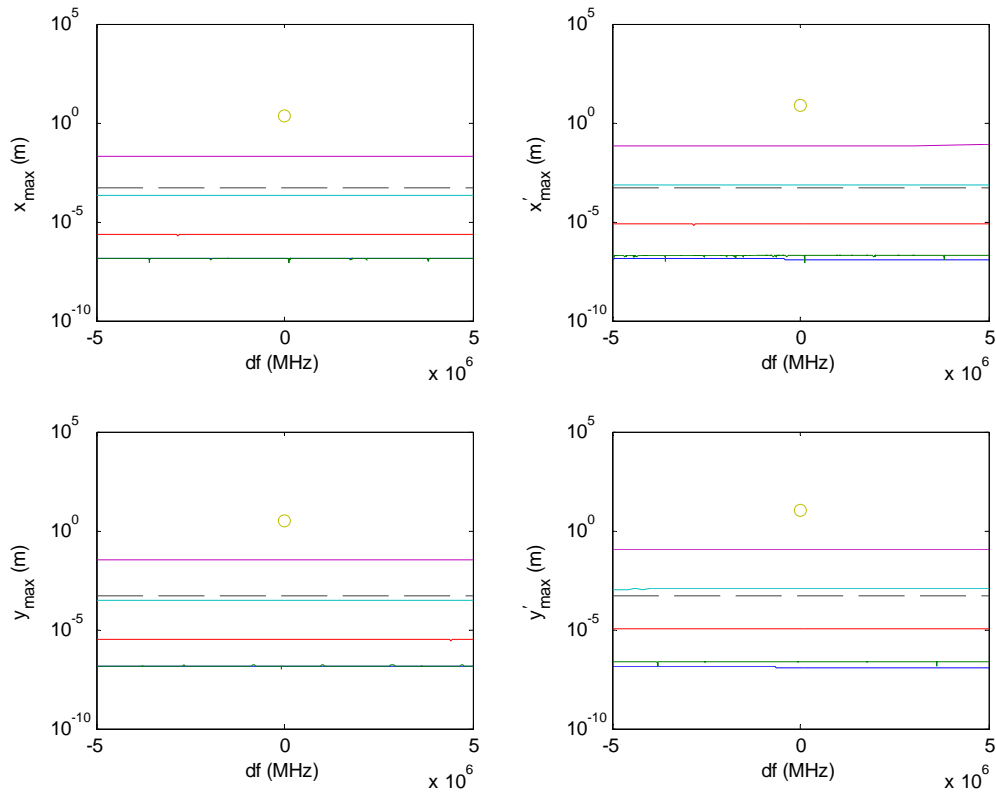


Figure 4: Maximum amplitude of oscillations during the transition as a function of the HOM frequency difference from its initial value. Frequency of corresponding HOMs in all the three cavities is the same. The bunch repetition rate is 748.5 MHz. Different lines correspond to different values of the HOM quality factor: blue corresponds to $Q=1.0e4$, green to $Q=1.0e5$, red to $Q=1.0e6$, light blue to $Q=1.0e7$, purple to $Q=1.0e8$, and the greenish circle corresponds to $Q=1.0e9$. The black dashed line shows $5.0e-4$, that corresponds to the maximum acceptable amplitude of oscillations.

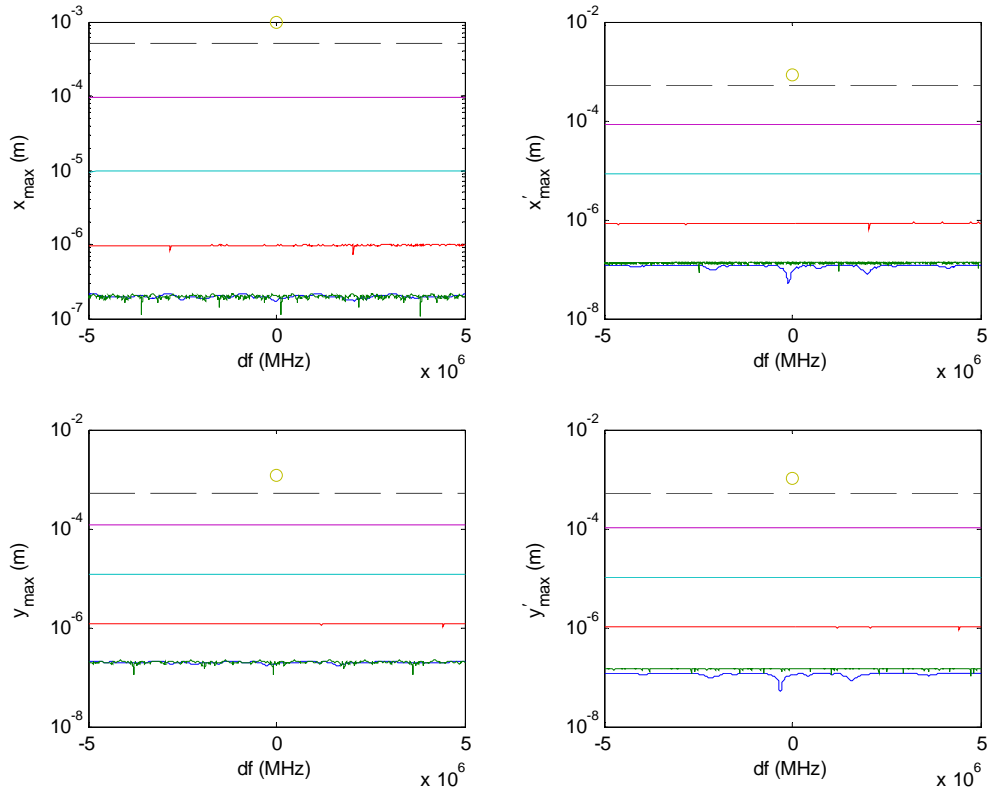


Figure 5: Maximum amplitude of oscillations during the transition as a function of the HOM frequency difference from its initial value. Frequency of corresponding HOMs in the first two cavities is the same. The frequency of HOMs in the third cavity is 10 MHz higher than the frequency of HOMs in the first two cavities. The bunch repetition rate is 748.5 MHz. Different lines correspond to different values of the HOM quality factor: blue corresponds to $Q=1.0e4$, green to $Q=1.0e5$, red to $Q=1.0e6$, light blue to $Q=1.0e7$, purple to $Q=1.0e8$, and the greenish circle corresponds to $Q=1.0e9$. The black dashed line shows $5.0e-4$, that corresponds to the maximum acceptable amplitude of oscillations.

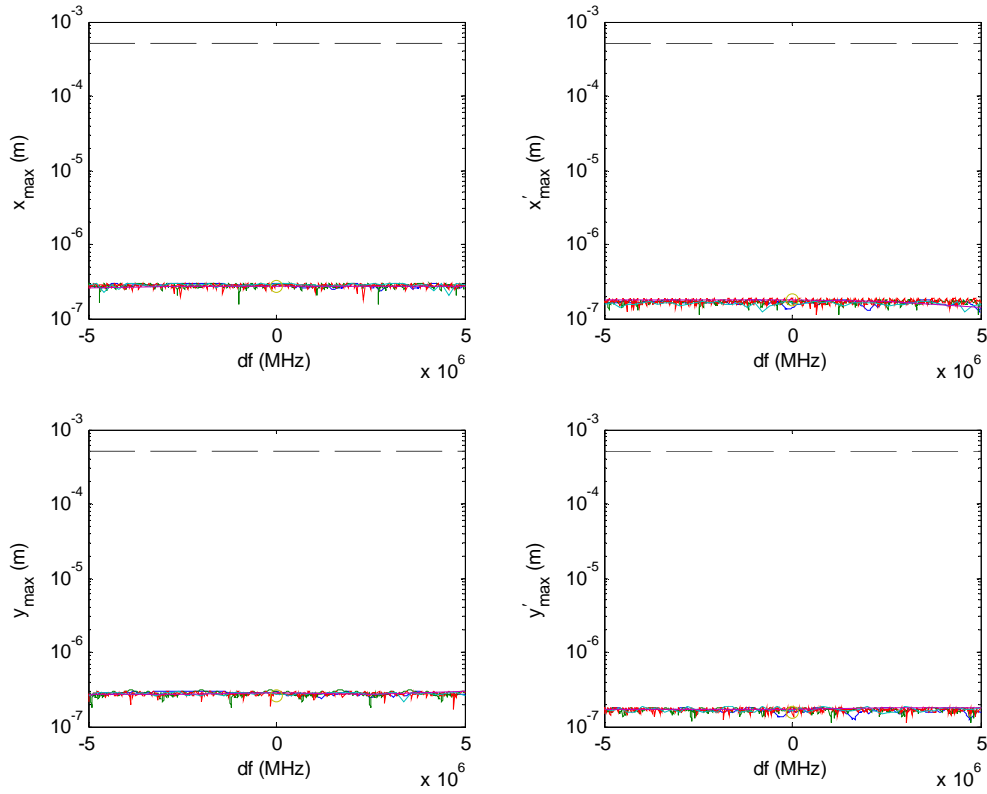


Figure 6: Maximum amplitude of oscillations during the transition as a function of the HOM frequency difference from its initial value. The frequency of HOMs in all the cavities is different. The bunch repetition rate is 748.5 MHz. Different lines correspond to different values of the HOM quality factor: blue corresponds to $Q=1.0e4$, green to $Q=1.0e5$, red to $Q=1.0e6$, light blue to $Q=1.0e7$, purple to $Q=1.0e8$, and the greenish circle corresponds to $Q=1.0e9$. The black dashed line shows $5.0e-4$, that corresponds to the maximum acceptable amplitude of oscillations.

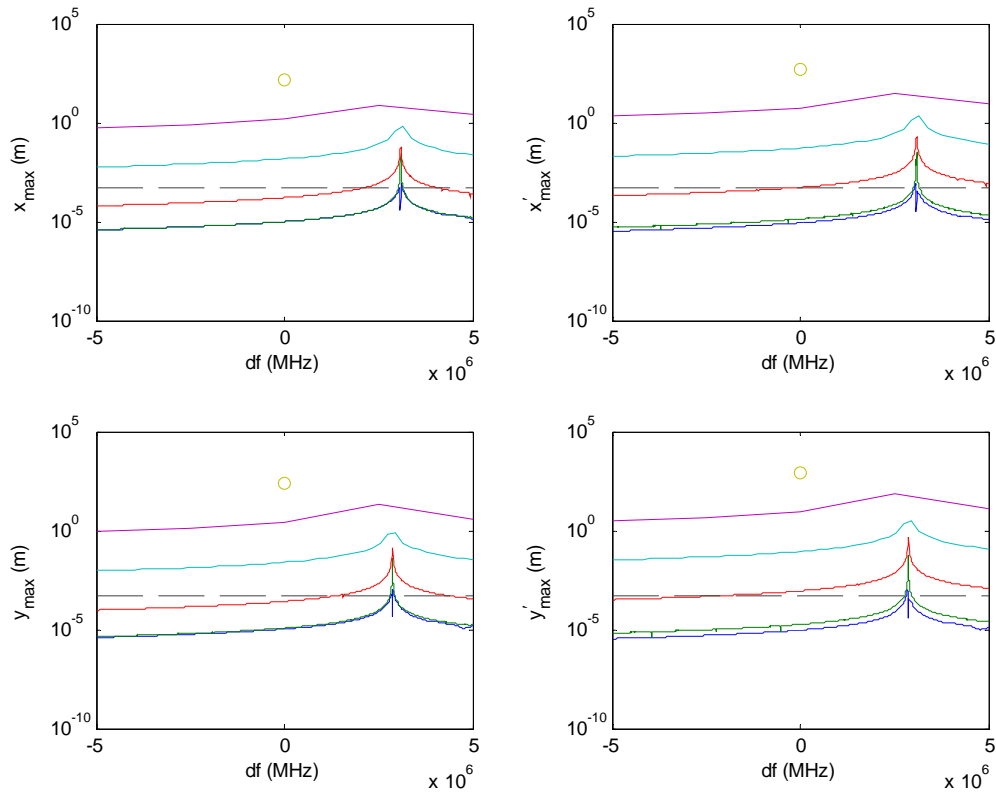


Figure 7: Maximum amplitude of oscillations during the transition as a function of the HOM frequency difference from its initial value. Frequency of corresponding HOMs in all the three cavities is the same. The bunch repetition rate is 74.85 MHz. Different lines correspond to different values of the HOM quality factor: blue corresponds to $Q=1.0e4$, green to $Q=1.0e5$, red to $Q=1.0e6$, light blue to $Q=1.0e7$, purple to $Q=1.0e8$, and the greenish circle corresponds to $Q=1.0e9$. The black dashed line shows $5.0e-4$, that corresponds to the maximum acceptable amplitude of oscillations.

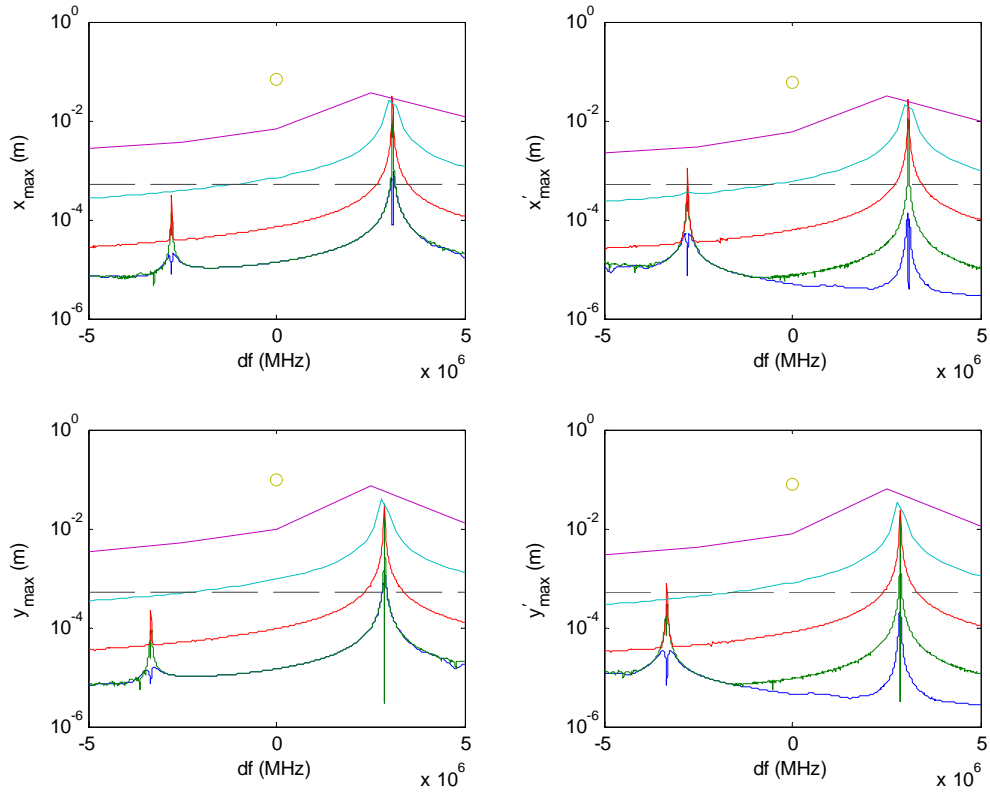


Figure 8: Maximum amplitude of oscillations during the transition as a function of the HOM frequency difference from its initial value. Frequency of corresponding HOMs in the first two cavities is the same. The frequency of HOMs in the third cavity is 10 MHz higher than the frequency of HOMs in the first two cavities. The bunch repetition rate is 74.85 MHz. Different lines correspond to different values of the HOM quality factor: blue corresponds to $Q=1.0e4$, green to $Q=1.0e5$, red to $Q=1.0e6$, light blue to $Q=1.0e7$, purple to $Q=1.0e8$, and the greenish circle corresponds to $Q=1.0e9$. The black dashed line shows $5.0e-4$, that corresponds to the maximum acceptable amplitude of oscillations.

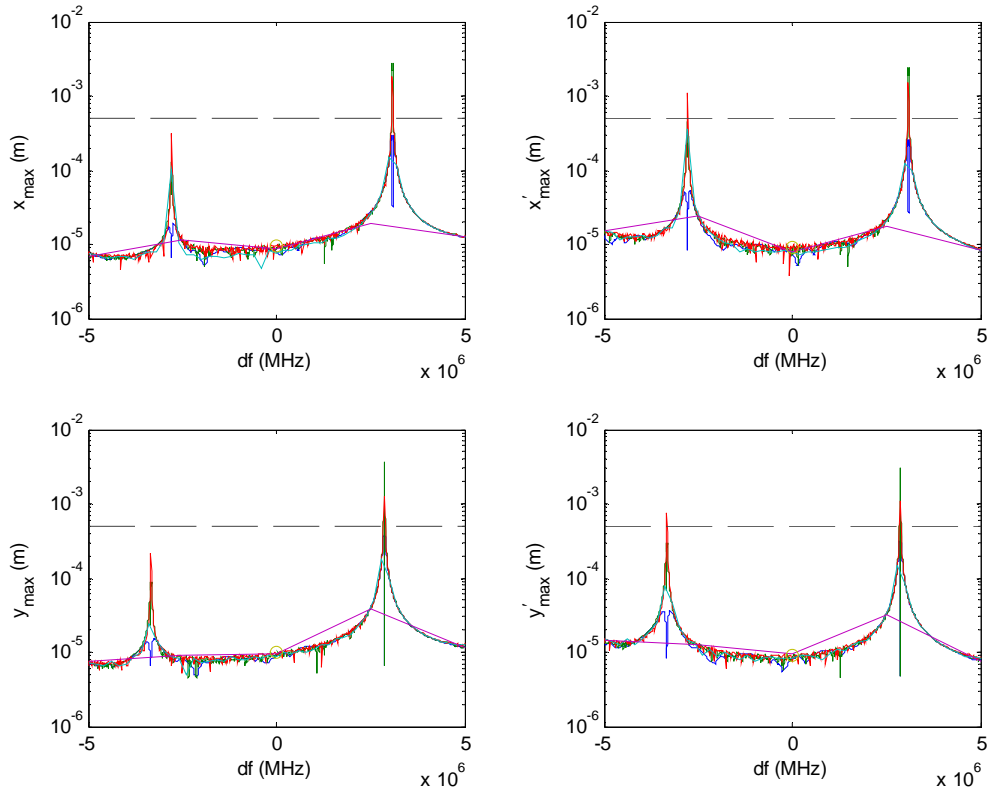


Figure 9: Maximum amplitude of oscillations during the transition as a function of the HOM frequency difference from its initial value. The frequency of HOMs in all the cavities is different. The bunch repetition rate is 74.85 MHz. Different lines correspond to different values of the HOM quality factor: blue corresponds to $Q=1.0e4$, green to $Q=1.0e5$, red to $Q=1.0e6$, light blue to $Q=1.0e7$, purple to $Q=1.0e8$, and the greenish circle corresponds to $Q=1.0e9$. The black dashed line shows $5.0e-4$, that corresponds to the maximum acceptable amplitude of oscillations.