Feasibility Study of Suppression of Multipass BBU at the JLab FEL by a Pulsed Phase Trombone.

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August 3, 2005

Abstract

It has been experimentally shown by Dave Douglas that a "static" phase trombone can stabilize a higher-order mode (HOM) or several modes at a time. However, the same phase trombone can "destabilize" other modes. At the 32$^{\text{nd}}$ ICFA Beam Dynamics Workshop on Energy Recovery Linacs, Reza Kazimi suggested to change the phase advance produced by the phase trombone randomly or using any other pattern to suppress the instability. In this paper, I analytically investigate the effect of the pulsed phase trombone on multipass BBU, assuming that the phase advance produced by the phase trombone periodically changes from 0 to π and back.

1 Introduction

The threshold of multipass BBU is given by

$$I_{\text{th}} = -\frac{2pc}{q\dfrac{\omega}{c}\left(\dfrac{R_d}{Q}\right)Q\,m_{12}\sin(\omega T_r)}. \tag{1}$$

If $m_{12}\sin(\omega T_r)$ is negative the mode can be unstable at a relatively low beam current. For the JLab FEL Upgrade this number is approx. 3 mA if no suppression technique is applied. If $m_{12}\sin(\omega T_r)$ is greater than zero, the threshold given by (1) is negative that implies absolute mode stability. However, accurate analysis and simulations show that the mode still can be unstable at a beam current of the order of $10^1 - 10^2$A. In this paper, I assume that, the mode is absolutely stable if $m_{12}\sin(\omega T_r) > 0$.

The product $m_{12}\sin(\omega T_r)$ will have a different sign for different modes, depending on each mode frequency and location in the machine. Thus, if a phase trombone is used to stabilize a particular mode, it can destabilize other modes, changing a sign of the product $m_{12}\sin(\omega T_r)$. The question brought up by Reza Kazimi at the ERL 2005 Workshop was: Can we overcome mulitpass BBU, if we will keep changing the phase advance produced by a phase trombone? In this paper, I analytically investigate the effect of a periodic variation of the phase advance by π.

2 Pulsed π-phase trombone

For now, let's consider a single HOM. The recirculation matrix for this HOM can be written as

$$M = CBA, \tag{2}$$

1

where A is the matrix from the HOM to the phase trombone, B is the matrix of the phase trombone, C is the matrix from the phase trombone back to the HOM. The m_{12} of the M is given by

$$m_{12} = c_{11}(b_{11}a_{12} + b_{12}a_{21}) + c_{12}(b_{13}a_{12} + b_{14}a_{22}) \tag{3}$$

The matrix elements b_{ij} are proportional to a combination of the sine and cosine of the phase advance of the phase trombone:

$$B = \begin{bmatrix} \dfrac{w_2}{w_1}\cos(\Delta\Psi) - w_1' w_2 \sin(\Delta\Psi) & w_1 w_2 \sin(\Delta\Psi) \\ -\left(\dfrac{1}{w_1 w_2} + w_1' w_2'\right)\sin(\Delta\Psi) + \left(\dfrac{w_2'}{w_1} - \dfrac{w_1'}{w_2}\right)\cos(\Delta\Psi) & \dfrac{w_1}{w_2}\cos(\Delta\Psi) + w_1 w_2' \sin(\Delta\Psi) \end{bmatrix}, \tag{4}$$

where $\delta\Psi$ is the phase advance produced by the trombone, w is related to the beta-function as $w = \sqrt{\beta}$, and subscripts 1 and 2 denote the entrance and the exit of the phase trombone. If the phase advance of the phase trombone is increased by π but the envelopes are kept the same, then the b_{ij}'s just change their sign. As the result, m_{12} changes its sign but not the magnitude. It is important to note that the sign of the m_{12} changes for every mode in every cavity. It follows from the fact that A and C are arbitrary.

3 HOM voltage evolution with the pulsed π phase trombone.

The HOM voltage amplitude behavior is described by

$$V = V_0 \exp\left(-t\frac{\omega}{2Q}\frac{I_{\text{th}} - I}{I_{\text{th}}}\right), \tag{5}$$

where the threshold current I_{th} is given by (1), I is the beam current. For a positive value of the threshold, the HOM voltage exponentially grows if $I > I_{\text{th}}$. If the m_{12} changes it sign, the threshold also changes its sign but not the absolute value. For a negative value of the threshold, the HOM voltage decays unless the beam current is too high and assumptions made during derivation of (1) and (5) fail [1].

Now let's assume that the additional phase advance produced by the phase trombone periodically changes from 0 to π and back with the time period $2T$. Assuming that the mode is unstable for the zero phase advance, I find that the voltage amplitude after $2T$ is given by

$$V = V_0 \cdot \exp\left(-T\frac{\omega}{2Q}\frac{I_{\text{th}} - I}{I_{\text{th}}}\right) \cdot \exp\left(-T\frac{\omega}{2Q}\frac{-I_{\text{th}} - I}{-I_{\text{th}}}\right) \tag{6}$$

which yields

$$V = V_0 \cdot \exp\left(-T\frac{\omega}{Q}\right). \tag{7}$$

One can see from the last equation that the HOM voltage decays with the characteristic time equal to $2Q/\omega$, the same as the natural voltage decay time. This is explained by the fact that the voltage decays faster than it grows for the same beam current but for opposite signs of the threshold.

4 Discussion

According to the simple analytical formulas presented in this paper, the pulsed π-phase trombone can be an effective tool for suppressing multipass BBU. It will work for all HOMs regardless their frequency and location in a machine.

However, there are several questions that have to be addressed in further studies:

- An important question is how fast and how frequently the phase has to be switched. According to the experimental data that Chris Tennant and I took at the FEL, it takes significantly longer for oscillations to grow up to a dangerous amplitude than the growth time because the initial amplitude is very small. For example, for a growth time of 2 msec or so, it was taking approx 20 msec after the beam was injected in the machine before a significant growth of the HOM voltage could be observed. This certainly gives extra time to switch. However, the growth time of the instability becomes shorter with the beam current. If the beam current exceeds the threshold by a factor of 10, the growth time will be approximately ten times shorter than the normal decay time. That means that the switching time has to be approximately 1 msec for the FEL, including the time when the instability grows but we still cannot see a significant effect.

- How switching the phase advance by π will affect the machine optics and beam losses.

- If the beam trajectory is offset from the axis of quadrupoles forming the phase trombone, variation of the quad strength will cause the beam trajectory to oscillate at the FEL. For FEL users, this will be equivalent to an increase of the effective beam emittance.

Analysis of other phase switching scenarios will require additional efforts and simulations.

References

[1] E. Pozdeyev, Phys. Rev. ST Accel. Beams, V8, 054401, 2005