α, β, ε at CEBAF - What has been measured? How well do others do?

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Abstract

Definitions of these beam parameters will be given. α and β measurements at the entrances to the experimental halls will be summarized, the latter normalized to design values. Measured emittances and their ratios to design values will be summarized. Best published values from KEKB and SLC will be summarized. Goals for CEBAF versus time will be suggested.

Definitions [1]

Equation of motion: u'' + k(s)*u = 0

where k(s) is a lattice function and u is either x or y. Let

$$u(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos(\Psi(s) - \Psi_0)$$

$$\Psi(s) = \int_0^s \frac{d\bar{s}}{\beta(s)} + \Psi_0$$

then (1) becomes

$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + \beta^2 k = 1$$

$$\alpha = -\frac{1}{2}\beta'$$

$$\gamma = (1+\alpha^2)/\beta$$

$$\beta'' + 2k\beta - 2\gamma = 0$$

$$\gamma u^2 + 2\alpha u u' + \beta u'^2 = \varepsilon$$

where the last is the Courant-Snyder invariant of motion and describes an ellipse of area πε.

The beam envelope is E(s) = $\pm\sqrt{\varepsilon}\sqrt{\beta(s)}$ where the $\pm$ indicates the envelope is on both sides of the beam centroid. This is what must fit in the pipe.

In CEBAF, the only dissipative mechanism which irreversibly increases emittance ε is due to synchrotron radiation; the term of art in accelerator physics is quantum excitations. Let η(s) be the dispersion function for the lattice. Define a lattice function H

$$H = \beta_x \eta'^2 + 2\alpha_x \eta\eta' + \gamma_x \eta^2$$

$$\frac{d\varepsilon_x}{ds} = \frac{55}{24\sqrt{3}} \frac{r_e hc}{2\pi mc^2} \gamma^5 \langle \frac{H}{\rho^3} \rangle_s$$

BPAM Results for α and β

The program BPAM was written by Y. Chao and is used in CEBAF to calculate beam parameters from data taken with multiple wire scanners in a line with a few quad settings instead of many quad settings with one wire scanner. The only documentation is the online manual [2].

Calculated α and β are shown in the table on the following page and in figures 1-4. Since the design beta at the start of each hall line is 10m for passes 1-4 and 20m for pass 5, the ratio of measured to design is shown for β. α is shown directly since all passes are designed to have zero alpha at the start of the hall lines. The spreadsheet accompanying this paper includes as well emittances, energy, momentum spread, "fractional χ^2" [2], and rms errors for beam parameters for 20 of the 36 lines, all the error values in the elogs. Five x pairs and two y pairs were removed from the table and figures due to large error bars. Elog numbers are included as well so those interested can review the circumstances of each measurement.

Clearly there are outliers in each plane in each of the figures. Unfortunately, the outliers are not coincident in data set across the four variables. There is a strong correlation between α_y and β_y, as seen in figure 5.

JMP output

All of the figures in this paper were produced with JMP, data exploration software from SAS. I've spent so much time with the program since 1991 that I forget that others might need an introduction. A reader of the second draft asked for one. Figure 1 is used as an example.

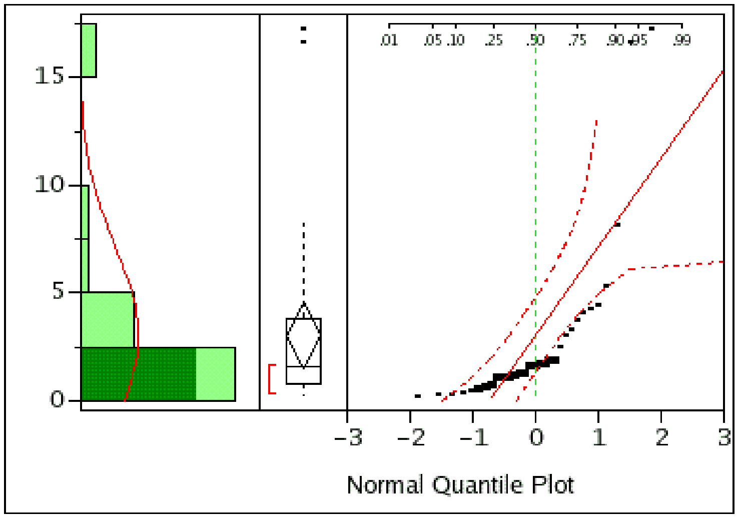
The type of graph, in this case a distribution, is given at the top. The variable being graphed is on the next line. Three graphs are given. Left to right, these are a simple histogram, a box-outlier plot, and a normal quantile plot. The histogram has the variable value on the Y axis and the number of counts on the unlabeled horizontal axis. Total number of observations is given as N in the Moments table. The horizontal lines in the box and ends of the diamond in the box-outlier plot label particular quantiles; I don't use it much. The normal quantile plot is defined so a normal distribution gives a straight line. The highlighted ranges in figures 1-4 were chosen to include straight portions of the normal quantile plots. Normal quantile plots in figures 2 and 4 suggest there are two normal distributions in alpha, one relatively tight in range [-1,1] and a much broader one with a larger slope. The quantile table gives the actual value in the data closest to the indicated percentage. The moments table gives moments. The "upper 95% Mean" and "lower 95% Mean" give the 95% confidence interval for the mean, roughly the mean plus and minus two times the standard error. No distributions are fitted to any of these data. Were one to take the highlighted subsets in figures 1-4 and replot, fitting normal distributions would be appropriate.

Figure 5 is a graph type admired by Edward Tufte. The variable names on the diagonal apply to the vertical axis when following a row and to the horizontal axis when following a column. The graphs are in 1:1 correspondence to the elements in the correlations matrix at the top of figure 5.

Date	hall	pass	betaX/design	alphaX	betaY/design	alphaY
07/27/05	A	3	0.49	-0.45	1.24	-0.43
07/19/05	A	3	1.14	0.26	0.37	0.75
07/19/05	A	3	1.09	-0.25	0.47	0.83
07/18/05	A	3	5.34	2.28		
07/18/05	A	3	1.92	4.02	3.94	0.78
07/17/05	A	3	1.62	3.72		
07/16/05	A	3	2.53	5.71	2.12	0.35
07/16/05	A	3	1.32	2.9	1.56	0.29
07/16/05	A	3	1.71	4.39	1.16	0.04
06/22/05	A	3	1.7	-0.02	0.83	0.15
06/04/05	A	4			2.12	-0.64
06/03/05	A	4			5.87	8.06
09/21/04	A	5	8.19	7.27	5.94	3.78
07/12/04	A	3	0.35	1.08	1.75	-0.7
06/24/04	A	3			1.45	-0.25
06/12/04	A	3	0.66	0.47	54.28	18.8
06/12/04	A	3	1.27	0.59	41.12	13.58
06/11/04	A	3			26.12	9.37
06/11/04	A	3	1.6	-0.93	21.08	7.73
05/05/04	A	5	0.61	3.84	0.45	0.67
04/20/04	A	5	0.18	1.01	1.54	-0.07
04/19/04	A	5	0.27	0.67	5.32	1.65
04/10/04	A	5	1.23	3.34	0.63	-1.28
04/07/04	A	5	0.4	0.14	3.05	1.67
01/28/04	A	4	4.28	1.71	0.75	0.15
01/26/04	A	4			2.92	1.89
01/21/04	A	4	3.01	0.03	18.59	7.91
07/15/03	A	2	1.14	0.19	0.92	0.08
06/20/05	B	5	16.7	0.84	1.25	1.74
06/20/05	B	5	17.27	0.88	1.03	1.54
06/20/05	B	5	4.49	-0.27	0.87	0.69
10/28/04	B	5	3.79	0.73	0.84	0.86
06/16/05	C	2	4.07	-8.41	0.8	-1.16
10/28/04	C	5	3.34	0.46	0.26	-0.58
04/28/04	C	4	0.77	-2.37	3.87	-5.7
11/22/03	C	3	1.94	-1.63	1.37	0.24

Distributions

Xbeta/design



Normal(3.04581,4.12434)

Quantiles

100.0%	maximum	17.270
99.5%		17.270
97.5%		17.270
90.0%		7.620
75.0%	quartile	3.790
50.0%	median	1.620
25.0%	quartile	0.770
10.0%		0.360
2.5%		0.180
0.5%		0.180
0.0%	minimum	0.180

Moments

Mean	3.0458065
Std Dev	4.124343
Std Err Mean	0.7407539
upper 95% Mean	4.5586277
lower 95% Mean	1.5329852
N	31

Fitted Normal

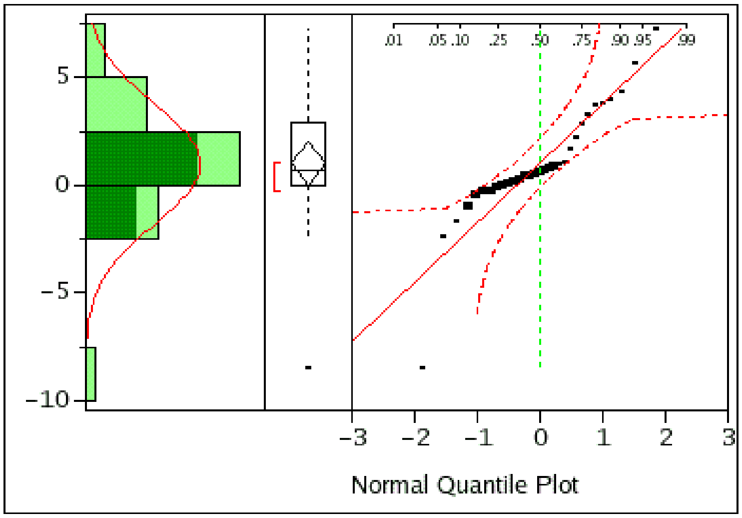
Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	Mu	3.045806	1.532985	4.558628
Dispersion	Sigma	4.124343	3.295812	5.512899

Figure 1. Distribution of ratios β_x measured/design. Fifteen of the values are in the range [0.5,2], 48%. These are highlighted. Five values excluded due to large error bars.

Distributions

X_alpha



Normal(1.03871,2.7657)

Quantiles

100.0%	maximum	7.270
99.5%		7.270
97.5%		7.270
90.0%		4.316
75.0%	quartile	2.900
50.0%	median	0.670
25.0%	quartile	-0.020
10.0%		-1.490
2.5%		-8.410
0.5%		-8.410
0.0%	minimum	-8.410

Moments

Mean	1.0387097
Std Dev	2.7657027
Std Err Mean	0.4967349
upper 95% Mean	2.0531776
lower 95% Mean	0.0242417
N	31

Fitted Normal

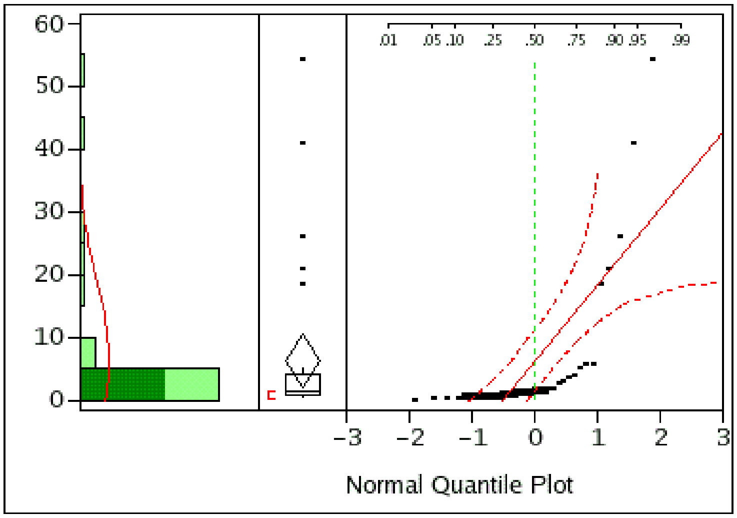
Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	Mu	1.038710	0.024242	2.053178
Dispersion	Sigma	2.765703	2.210106	3.696841

Figure 2. Distribution of measured α_x values. Sixteen are within range [-1,1]. In only 7 of 31 likely valid measurements is this criterion satisfied simultaneously with the beta condition in figure 1. Mean and median are rather far from design value zero. Mean falls outside the range of interest.

Distributions

Ybeta/design



Normal(6.34941,12.2061)

Quantiles

100.0%	maximum	54.280
99.5%		54.280
97.5%		54.280
90.0%		23.600
75.0%	quartile	4.285
50.0%	median	1.495
25.0%	quartile	0.837
10.0%		0.460
2.5%		0.260
0.5%		0.260
0.0%	minimum	0.260

Moments

Mean	6.3494118
Std Dev	12.206072
Std Err Mean	2.0933241
upper 95% Mean	10.608312
lower 95% Mean	2.0905118
N	34

Fitted Normal

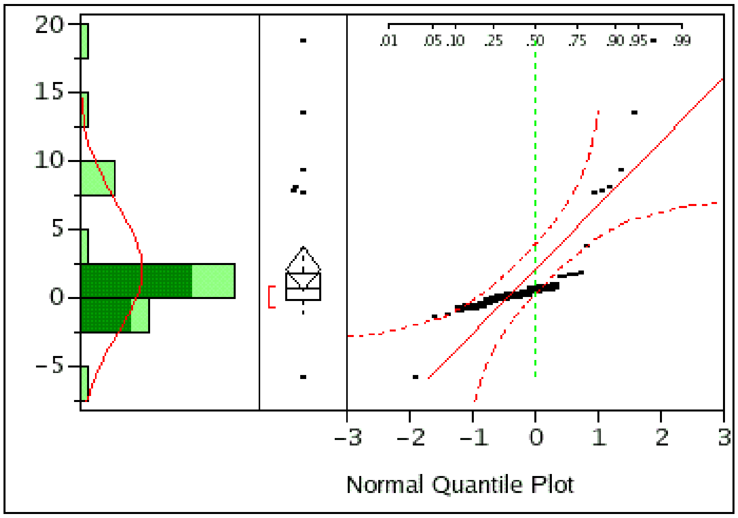
Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	Mu	6.34941	2.090512	10.60831
Dispersion	Sigma	12.20607	9.845132	16.06658

Figure 3. Distribution of ratios β_y measured/design. Sixteen of the values are in the range [0.5,2], 47%

Distributions

Y_alpha



Normal(2.14088,4.69261)

Quantiles

100.0%	maximum	18.80
99.5%		18.80
97.5%		18.80
90.0%		8.71
75.0%	quartile	1.78
50.0%	median	0.68
25.0%	quartile	-0.12
10.0%		-0.93
2.5%		-5.70
0.5%		-5.70
0.0%	minimum	-5.70

Moments

Mean	2.1408824
Std Dev	4.6926056
Std Err Mean	0.8047752
upper 95% Mean	3.7782099
lower 95% Mean	0.5035548
N	34

Fitted Normal

Parameter Estimates

Type	Parameter	Estimate	Lower 95%	Upper 95%
Location	Mu	2.140882	0.503555	3.778210
Dispersion	Sigma	4.692606	3.784946	6.176772

Figure 4. Distribution of measured α_y. Nineteen are in range [-1,1]. These are highlighted. In twelve cases this constraint and that in figure 3 are simultaneously satisfied.

Multivariate

Correlations

	X_alpha	Y_alpha	Xbeta/design	Ybeta/design
X_alpha	1.0000	0.0527	0.0641	-0.0528
Y_alpha	0.0527	1.0000	-0.0190	0.9449
Xbeta/design	0.0641	-0.0190	1.0000	-0.1496
Ybeta/design	-0.0528	0.9449	-0.1496	1.0000

7 rows not used due to missing values.

Scatterplot Matrix

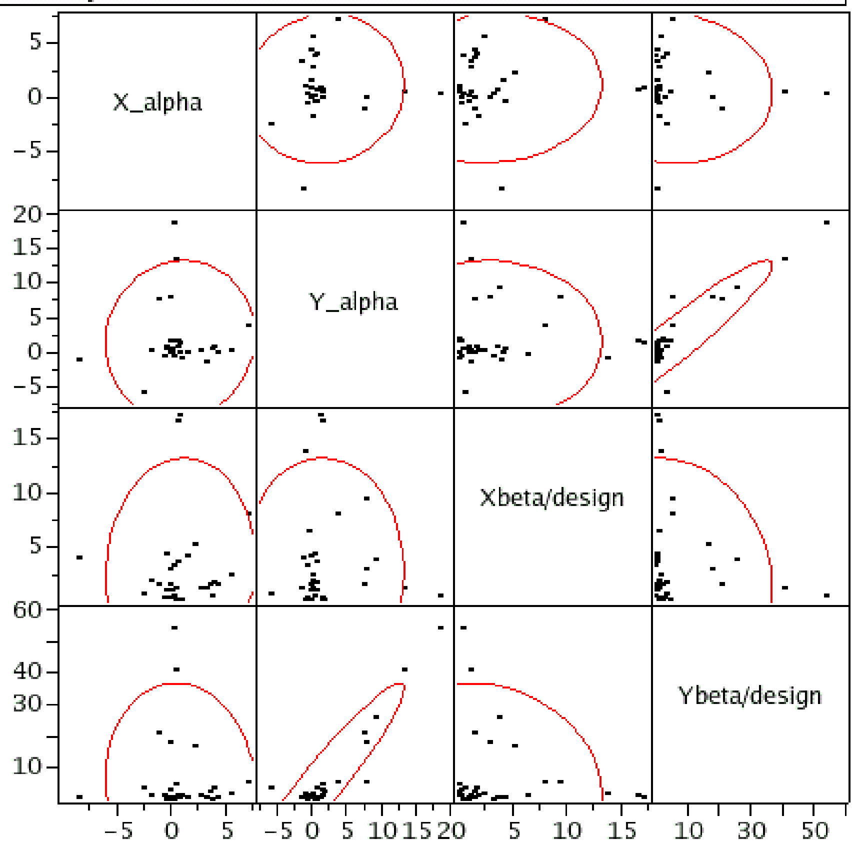


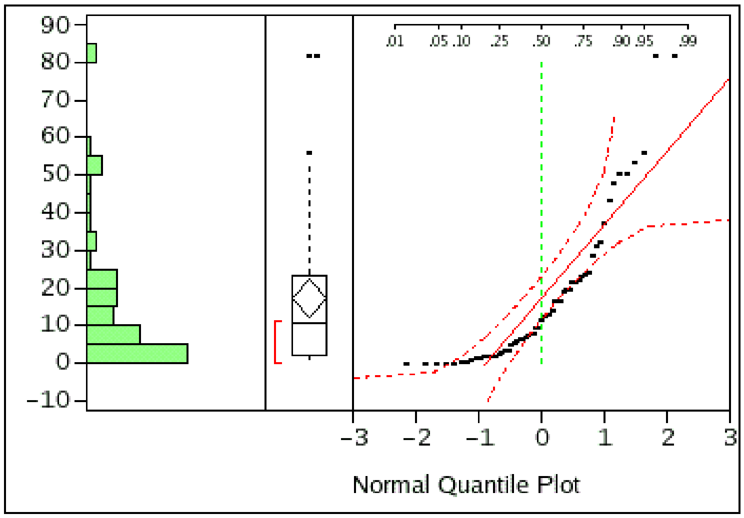
Figure 5. Correlations among four variables in measured data. Correlation coefficient is 0.9 between α_y and β_y but only -0.06 between α_x and β_x. Seven rows with large error bars on either x or y values were not used.

BPAM results for emittance

BPAM also calculates values for geometric emittance. In the literature, emittance normalized to energy is generally used because it's supposed to be constant. In the two graphs that follow I further normalize normalized emittance by dividing it by the value at 100 keV (0I05 harp) for typical beams (0.05 mm-mr) or G0/HAPPEX-He beam (0.15 mm mr). These can be taken as design values for normalized emittance for the machine. In other words, the graphs following show normalized emittance divided by design. The first two figures show all measurements.

Distributions

X_norm_emit/design



Quantiles

100.0%	maximum	81.577
99.5%		81.577
97.5%		81.521
90.0%		50.197
75.0%	quartile	23.247
50.0%	median	10.493
25.0%	quartile	2.104
10.0%		0.162
2.5%		0.000
0.5%		0.000
0.0%	minimum	0.000

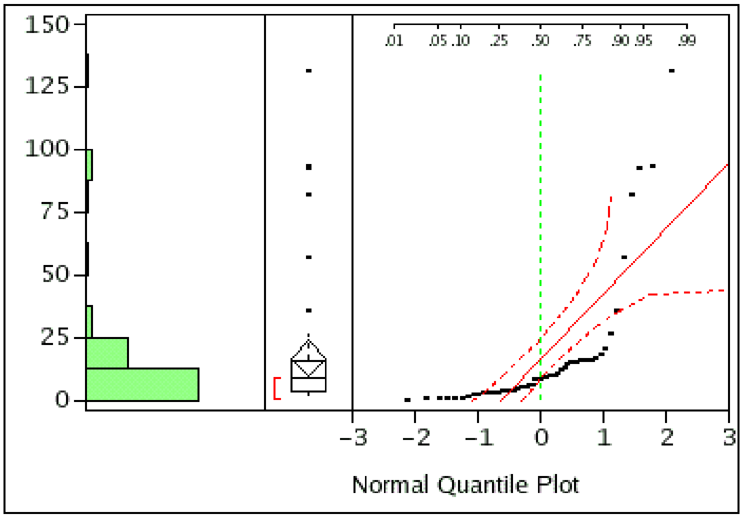
Moments

Mean	17.27464
Std Dev	19.573038
Std Err Mean	2.5700659
upper 95% Mean	22.421108
lower 95% Mean	12.128172
N	58

Figure 6. X normalized emittance measurements divided by design value.

Distributions

Y_norm_emit/design



Quantiles

100.0%	maximum	131.45
99.5%		131.45
97.5%		117.36
90.0%		46.71
75.0%	quartile	16.16
50.0%	median	8.92
25.0%	quartile	3.60
10.0%		1.45
2.5%		0.76
0.5%		0.61
0.0%	minimum	0.61

Moments

Mean	16.734601
Std Dev	26.161543
Std Err Mean	3.5601351
upper 95% Mean	23.875328
lower 95% Mean	9.5938746
N	54

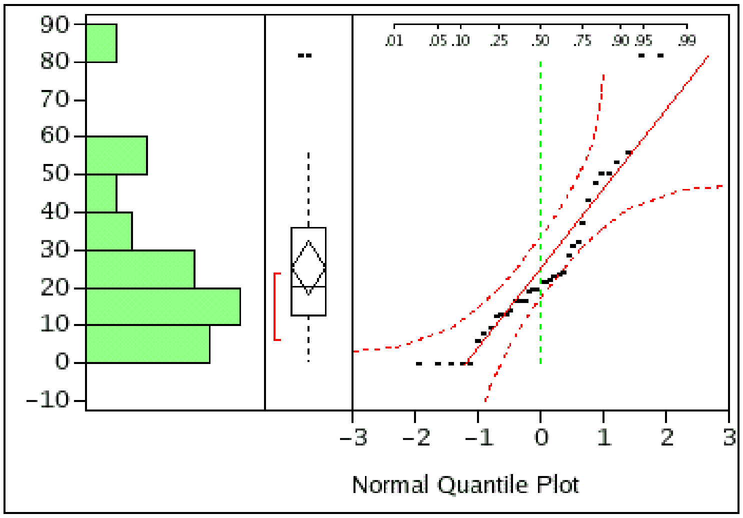
Figure 7. Y normalized emittance measurements divided by design value

As mentioned above, normalized emittance is supposed to be constant across the machine for CEBAF at/below 6 GeV - quantum excitation is not a significant effect. It is very far from constant. Given that normalized emittance increases through the machine, the inclusion of injector measurements in figures 6 and 7 distorts the situation. They are removed in the next two figures.

The next two graphs might be thought to continue to distort things as they contain 2 measurements from pass two beam, 17 from pass three, 6 from pass four and 11 from pass five. However, there is no correlation between the values in the next two figures and pass number, so it's appropriate to simply present the distributions. This lack of correlation and observations not presented here suggest that most of the emittance growth in CEBAF is due to x-y coupling in the injector and betatron mismatch between the injector and North Linac. x-y coupling may also be introduced by cavity gradient calibration errors and resulting errors in compensating skew quad settings, but these are observed to be small starting after the first pass through the North Linac. Thus most of the emittance growth is thought to originate in the first 500m of the machine. This is consistent with the lack of correlation of normalized emittance with pass number.

Distributions

X_norm_emit/design



Quantiles

100.0%	maximum	81.577
99.5%		81.577
97.5%		81.577
90.0%		54.033
75.0%	quartile	35.886
50.0%	median	20.518
25.0%	quartile	12.561
10.0%		0.000
2.5%		0.000
0.5%		0.000
0.0%	minimum	0.000

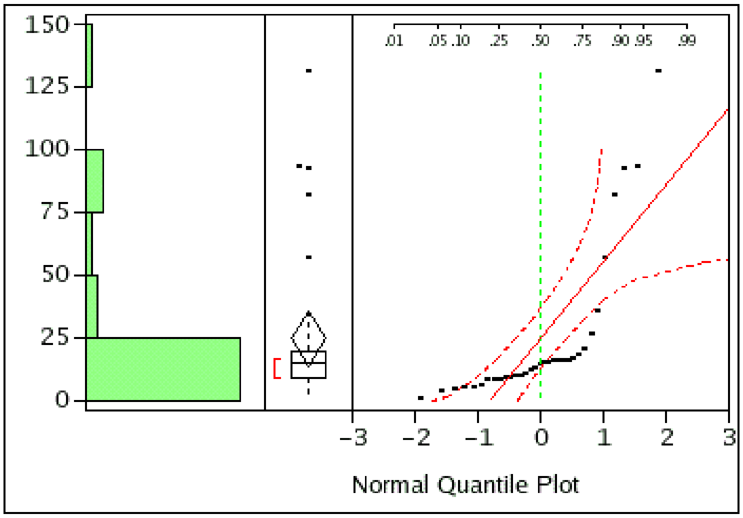
Moments

Mean	25.338249
Std Dev	21.017925
Std Err Mean	3.5029876
upper 95% Mean	32.449692
lower 95% Mean	18.226806
N	36

Figure 8. X normalized emittance divided by design, measurements done in the halls

Distributions

Y_norm_emit/design



Quantiles

100.0%	maximum	131.45
99.5%		131.45
97.5%		131.45
90.0%		88.42
75.0%	quartile	19.81
50.0%	median	15.06
25.0%	quartile	8.92
10.0%		5.31
2.5%		1.43
0.5%		1.43
0.0%	minimum	1.43

Moments

Mean	24.858935
Std Dev	30.836345
Std Err Mean	5.3679187
upper 95% Mean	35.793028
lower 95% Mean	13.924843
N	33

Figure 9. Y normalized emittance divided by design, measurements done in the halls

Speculation on causes of emittance growth

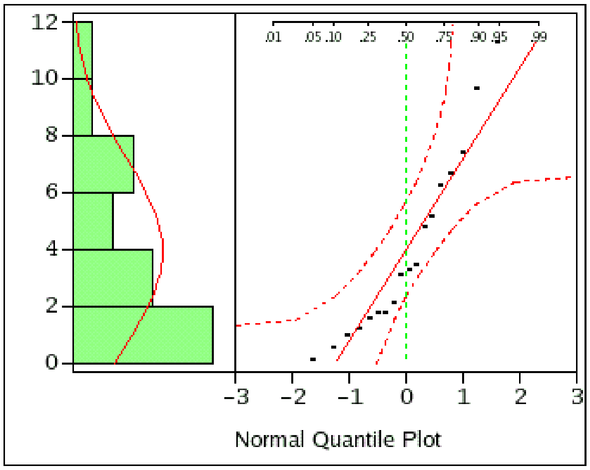
There are three causes of emittance growth thought relevant to CEBAF:

1. quantum excitations, aka growth driven by synchrotron radiation
2. x-y coupling in injector and North Linac
3. betatron mismatch, principally at the interfaces between the linacs and arcs, aka the spreaders and recombiners

The first is inevitable; the second and third are not. In figure 10 emittance growth through the end of the 5 MeV region is shown. Both distibutions are consistent with normality.

Distributions

X_norm_emit/design



Normal(3.97827,3.20633)

Quantiles

100.0%	maximum	11.291
99.5%		11.291
97.5%		11.291
90.0%		9.854
75.0%	quartile	6.371
50.0%	median	3.222
25.0%	quartile	1.503
10.0%		0.561
2.5%		0.180
0.5%		0.180
0.0%	minimum	0.180

Moments

Mean	3.9782696
Std Dev	3.2063281
Std Err Mean	0.7557388
upper 95% Mean	5.5727391
lower 95% Mean	2.3838001
N	18

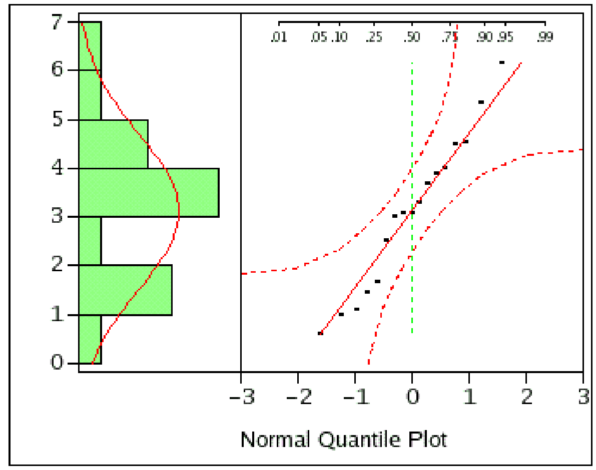
Fitted Normal

Goodness-of-Fit Test

Shapiro-Wilk W Test

W	Prob<W
0.907906	0.0790

Y_norm_emit/design



Normal(3.12903,1.57683)

Quantiles

100.0%	maximum	6.1516
99.5%		6.1516
97.5%		6.1516
90.0%		5.5286
75.0%	quartile	4.2633
50.0%	median	3.0953
25.0%	quartile	1.5866
10.0%		0.9325
2.5%		0.6132
0.5%		0.6132
0.0%	minimum	0.6132

Moments

Mean	3.1290345
Std Dev	1.5768277
Std Err Mean	0.3824369
upper 95% Mean	3.9397645
lower 95% Mean	2.3183045
N	17

Fitted Normal

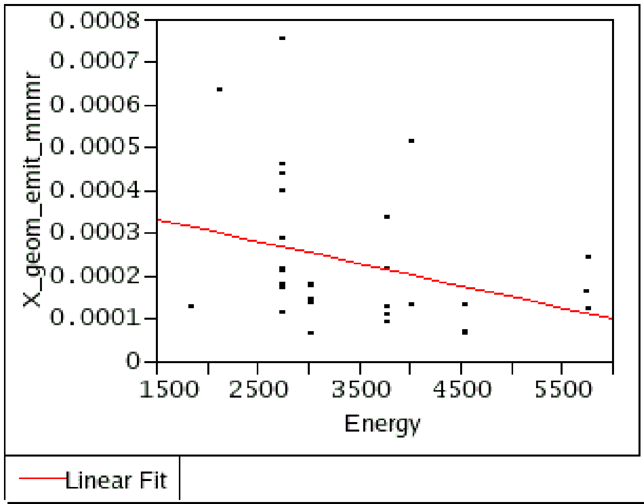
Goodness-of-Fit Test

Shapiro-Wilk W Test

W	Prob<W
0.969168	0.8037

Figure 10. Distributions of doubly normalized emittances at 0L04 (5 MeV). If the values were all unity, normalized emittance would have been constant as desired in an accelerator. The unphysically low minimum in the X data is a measurement in the 5 MeV region. The next lowest X value is 0.6 and corresponds to the minimum Y value of 0.6. If I got the laser spot wrong in mormalizing , the 0.6 values would be 1.8, closer to the center of the distributions.

Bivariate Fit of X_geom_emit_mmmr By Energy



Linear Fit

X_geom_emit_mmmr = 0.0004141 - 5.1767e-8 Energy

Summary of Fit

RSquare	0.094649
RSquare Adj	0.06343
Root Mean Square Error	0.000166
Mean of Response	0.000235
Observations (or Sum Wgts)	31

Analysis of Variance

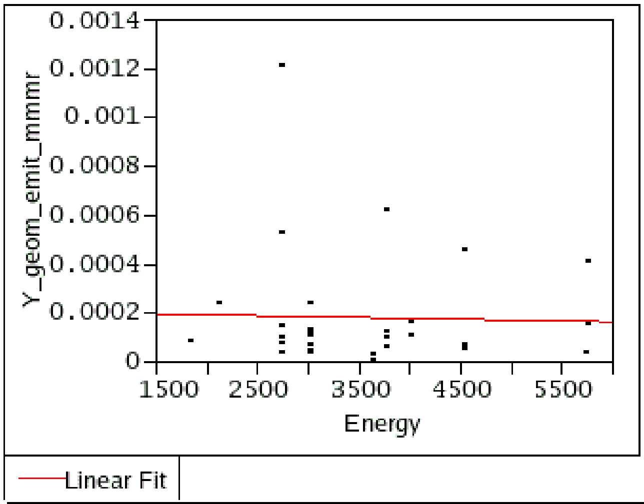
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	8.31472e-8	8.3147e-8	3.0318
Error	29	7.95328e-7	2.7425e-8	Prob > F
C. Total	30	8.78476e-7		0.0922

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0004141	0.000107	3.86	0.0006
Energy	-5.177e-8	2.973e-8	-1.74	0.0922

Figure 11 X geometric emittance (mm-mrad) measured in the halls as a function of energy. The energy dependence is NOT significant at the P=0.05 level as the F ratio is under 4, as the "Prob > F" value of 0.0922 in the table indicates. Similar lack of significance is seen if one plots this variable or the corresponding normalized emittance against pass number. Thus distance traveled by the beam, at least for passes 2-5, doesn't appear to change emittance.

Bivariate Fit of Y_geom_emit_mmmr By Energy



Linear Fit

Y_geom_emit_mmmr = 0.0002113 - 6.7141e-9 Energy

Summary of Fit

RSquare	0.000767
RSquare Adj	-0.03147
Root Mean Square Error	0.00024
Mean of Response	0.000188
Observations (or Sum Wgts)	33

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	1.37214e-9	1.3721e-9	0.0238
Error	31	0.00000179	5.763e-8	Prob > F
C. Total	32	0.00000179		0.8784

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0002113	0.000158	1.34	0.1902
Energy	-6.714e-9	4.351e-8	-0.15	0.8784

Figure 12. Y geometric emittance (mm-mrad) measured in the halls as a function of energy. No dependence at all on energy or pass number 2-5. Eliminated the outlier at the top doesn't change the validity of this statement.

As mentioned just before figure 8, it is not known how much of the emittance growth is due to x-y coupling and how much to betatron mismatch. There are strongly held opinions; I'm agnostic. It is known that most of the growth is in the first 500 m of the machine, before the 1S region where 30 Hz corrector excitation for "Courant-Snyder" matching begins. A hardware change which will help distinguish among the two hypotheses is discussed below in the section on meeting proposed goals.

Comparison with other labs

A literature search was done to learn how well normalized emittance is held in other machines. It was soon learned that proton accelerator complexes are not relevant for comparison because collimators are placed in transfer lines to ensure that the beam coming out of one machine fits in the acceptance aperture of the next ring in the sequence at the latter's injection energy. Often 20% or more of the beam is scraped off. A paper [3] examining placement of wire scanners in one of the SSC transfer lines looks at achievable measurement error and the emittance growth the resulting mismatch may cause. Best case found was 6% growth with 20% measurement error. In the hall lines BPAM reports ~15% measurement error for β and ~5% for α using more wire scanners than assumed in [3]. Beyond this, emittance values for SLC at SLAC and KEKB electron machines were the only ones found that are useful for comparison.

KEKB [ref 4, figure 3]
19 to 1500 MeV in linac, 1 nC bunch
horizontal: factor of three growth in normalized emittance
vertical: factor of six

SLC [ref 5, caption of figure 1]
At entrance to damping rings, three times design, but not a lot of care taken as damping rings define emittance for the rest of the system. Damping rings at 1.15 GeV

SLC exit of damping rings to interaction point (1.15 to 47 GeV) [ref 5, figure 9]
February 88 through August 89, measured/design improved as follows
X from 6.5:1 to 1.3:1
Y from 4.5:1 to 1.3:1

SLC 1997 run [ref 6, table 1]

location	ε_x ***10^{-5}m***	***ratio***	ε_y ***10^{-5}m***	***ratio***
linac start	3.5		0.5	
linac end	4.5	1.3	0.9	1.8
interaction point	5.3	1.5	1.3	2.6
design at IP	3	IP: 1.8 design	3	IP 0.4 design

For CEBAF measurements in the halls, in only three of 36 cases was the ratio of normalized emittance to design less than ten in both planes. All of the values in these pairs were greater than five. Only one of the Y values was less than two and the error bar on the associated X value was so great as to exclude it from the set, suggesting the Y value is also questionable.

α and β comparisons with other labs aren't shown because they all have sufficient degrees of freedom immediately before their interaction points to set these variables as they wish. CEBAF does not have the four needed degrees of freedom (DOF) in halls B and C but space is available to add the one DOF needed in each. Hall A has sufficient DOF if the Moller quads are unconstrained but only two DOF if Moller quads are set for Moller measurements. Space is not available in hall A to improve this situation. [7]

Possible goals for CEBAF

The author suggests the following goals based on this work. All of the emittance goals will likely require the installation of wire scanners in arcs. Wire scanners originally installed there were moved to the hall lines a few years ago to assist in betatron matching using BPAM.

for all setups after Jan. 1, 2007

α within range [-1, 1] at entrance to hall lines A, B and C
β/design within range [0.5,2] at entrance to hall lines A, B and C
ε/design < 10 at entrance to hall lines A, B and C

with additional instrumentation and/or injector rework (mid-2008??)

α within range [-0.5, 0.5] at entrance to hall lines A, B and C
β/design within range [0.5, 1.5] at entrance to hall lines A, B and C
ε/design < 3 at entrance to hall lines A, B and C

2010

α within range [-0.5, 0.5] at entrance to hall lines A, B and C
β/design within range [0.5, 1.5] at entrance to hall lines A, B and C
ε/design <2 at entrance to hall lines A, B and C

What is needed to reach these goals?

The principal reason for this tech note is to stimulate effort to answer this question. A few comments will be made now:

During the first week of October 2005 two additional skew quads will be installed in the 5 MeV region of the injector. This will provide four normal and four skew quads to deal with phase space from the 0I05 BPM through the middle of the 5 MeV region. This region is rarely changed and it is hoped that once a new solution with minimum coupling is developed it will remain unchanged forever. After the installation there will also be four normal and four skew quads to deal with coupling and emittance growth from 5 MeV through the two full cryomodules in the injector to the end of the 0L06 girder. Normal quads 0L07-0L10 will then be left to match the resulting beam into the North Linac. Should skews be added to 0L07-10 or in the chicane?

There is a harp at the end of the 1E03 girder, just before arc 1, which may be used to measure emittance growth from the injector through the first pass in the North Linac. There is another at 2E03 to check first pass through the South Linac.

The only skew quads unassociated with the 5 MeV region or cryomodules are one at 8S01 and one at 9S02. There's room for groups of four skew quads to manipulate phase space in 3E, 5E, 7E and 9E as these are symmetric with the extraction regions on the west end. One might take a page from the hadron accelerator handbook and install a collimator system in 9E instead.

Installation of additional diagnostics, magnets and collimators are among the topics that should be discussed in answering the question asked in the section head.

Summary

α and β measurements at the entrances to the experimental halls were summarized. Ratios of measured to design emittances were summarized, for all measurements including injector and the halls alone. Best published values from KEKB and SLC were provided. Goals for CEBAF versus time were suggested.

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References

1. Particle Accelerator Physics vol I and II, Helmut Wiedemann, Springer 1995

2. Beam Profile Analysis and Matching (BPAM) program user guide
http://opsntsrv.acc.jlab.org/ops_docs/online_document_files/MCC_online_files/BPAM_automatch_user_guide.pdf

3. OPTIMAL PLACEMENT OF PROFILE MONITORS IN A MISMATCHED FODO LATTICE* K. Bertsche, Fermi National Accelerator Laboratory † , P. O. Box 500, Batavia, IL ...*http://accelconf.web.cern.ch/AccelConf/p95/ARTICLES/MPQ/MPQ21.PDF - 35.0KB*

4. BEAM OPTICS MATCHING IN THE KEKB INJECTOR LINAC; T. Kamitani, H. Koiso, N. Akasaka, A. Enomoto, J. Flanagan, H. Fukuma, Y. Funakoshi, K. Furukawa, N. Iida, T. Ieiri, T. Nakamura, Y. Ogawa, S. Ohsawa, K. Oide, K.
.*http://accelconf.web.cern.ch/AccelConf/a98/APAC98/5D019.PDF - 243.8KB*

5. Summary of Emittance Control in the SLC Linac* J. T. Seeman, C. Adolphsen, K. L. F. Bane, P. Emma, F. J. Decker, I. Hsu, T. Limberg, L. Merminga, M. Ross, and W.
...*http://accelconf.web.cern.ch/AccelConf/p91/PDF/PAC1991_2064.PDF - 302.1KB*

6 Accelerator Physics Highlights in the 1997/98 SLC Run , R Assmann, KLF Bane, T Barklow, JR Bogart, Y Cai et al. , Asian Particle Accelerator Conference (APAC 98), Tsukuba, 1998
http://epaper.kek.jp/a98/APAC98/5D034.PDF

7. Exploration of robust matching alternatives for hall lines using B as a test bed J. Benesch JLAB TN 03-037 http://tnweb.jlab.org/tn/2003/03-037.pdf

Associated spreadsheet mentioned on page 2:

J:\Office_files\12GeV
BPAM_result_summar