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On the Luminosity Advantage of ERLs for Collider Applications

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Recently the CASA group has been exploring ring-ring colliders for ELIC applications. During the course of the studies, it was noticed that the luminosity penalty for replacing the energy recovery linac for electrons by the storage ring was not as great as might have been expected, *a priori*. In fact, for ELIC parameters, with the electron optics and ion ring optics identical between the two cases, the difference in ultimate luminosity is less than a factor of two. Simple luminosity scaling formulas can explain this somewhat surprising result. They also demonstrate why the case for an ERL-ring collider is much clearer for eRHIC parameters.

For many years it has been known that it is useful for predicting luminosity of storage ring colliders to express the luminosity in terms of the beam-beam tune shift. The luminosity for a collider designed to have the same *rms* spot sizes in the two transverse directions of the two colliding beams is

$$L = \frac{fN_e N_i}{4\pi\sigma_x\sigma_y},$$

where f is the collision frequency, N_e and N_i are the number of electrons and ions per bunch, and σ_x and σ_y denote the common *rms* spot size at collision. On the other hand, the beam-beam tune shift for the ions in the two transverse directions is

$$\xi_x^i = \frac{N_e r_i}{2\pi\gamma_i}\frac{1}{\varepsilon_x^i\left(1+\sigma_y/\sigma_x\right)}$$

$$\xi_y^i = \frac{N_e r_i}{2\pi\gamma_i}\frac{1}{\varepsilon_y^i\left(1+\sigma_y/\sigma_x\right)\sigma_x/\sigma_y},$$

where r_i is the classical ion radius, γ_i is the ion energy, and ε^i is the ion emittance. Clearly, the luminosity can be written in terms of the (larger) vertical tune shift as

$$L = \frac{fN_i\xi_y^i\gamma_i}{2r_i\beta_{i,y}^*}\left(1+\sigma_y/\sigma_x\right), \tag{1}$$

which gives the luminosity in terms of the ion current, ion energy, ion beam-beam tune shift, ion vertical beta function at the collision point, and the beam aspect ratio at the collision point. Now the argument comparing the ring-ring and ERL-ring collider can follow that in the annual review article [1].

As a *Gedanken* experiment, suppose that one has a stable ring-ring collider design. Because the ions must remain stable in the ring in going from a ring-ring collider to an ERL-ring collider, there isn't much possibility to change the beam-beam parameter up (in fact most of our parameter lists have assumed the ion beam-beam parameters are at the level of performance in the best present colliders). If we compare colliders at the same ion energy, γ_i cannot change, and r_i is a fundamental constant. Luminosity gains must arise from the possibility of increasing the ion current fN_i circulating in the ring, or from decreasing the ion vertical beta-function at the interaction point. If the parameters in the ion ring are already chosen so that there is little possibility to further increase the current in going to the ERL-ring design, e.g., the ion current is already limited by some non-beam-beam effect such as the Lasslett tune shift or IBS heating that can no longer be electron cooled, there will not be any possibility to increase the luminosity in an ERL-ring design compared to a ring-ring design from the current factor. Likewise, a similar argument shows that unless there is some advantage to the ERL-ring design that allows one to design in smaller vertical beta-functions for the ions at collision, again, there will be no inherent advantage of the ERL-ring design.

One principal advantage of the ERL accelerator for the electrons is that the emerging emittance is expected to be lower than that of a similar current stored in a storage ring. Because in a collider the electron spots cannot be made arbitrarily smaller, to take advantage of the better emittance, due to the non-linear beam-beam effect, and because, unfortunately, for collider applications where the ion beam-beam tune shift is the most important limit to satisfy in order to allow stable motion in the ion ring, the luminosity no longer depends directly on the electron emittance. The principal advantage of the ERL-ring collider has limited value to increase collider luminosity in many parameter regimes.

Table 1 shows the luminosity as calculated by Eqn. (1) for the old ERL-ring ELIC [2] and the new ring-ring designs of Derbenev and Yunn at 150 × 7 GeV, for the parameters in the tables. As one can see, at ELIC there will be relatively little luminosity lost in going to a ring-ring design because the ion ring current, $\beta^*_{i,y}$, and beam-beam tune shift are similar. Yunn's parameters are somewhat more conservative as $\beta^*_{i,y}$ is a factor of 4 bigger.

	ERL-ring	Derbenev ring-ring	Yunn ring-ring
I_i (A)	1.0	1.0	1.0
ξ^i_y	0.01	0.0065	0.0065
$\beta^*_{i,y}$ (cm)	0.5	0.5	2.0
σ_y / σ_x	0.2	0.2	0.2
L (standard units)	7.8×10^{34}	5.1×10^{34}	1.27×10^{34}

Table 1: Comparison ERL-ring and ring-ring interaction point luminosities

Finally, it is worth taking a look at the eRHIC parameters to see whether there is a more compelling argument for the ERL-ring design there. Table 2 shows the comparison, as shown in the Brookhaven ZDR for 250 × 10 GeV collisions [3]. The two cases in the ERL-ring column correspond to simultaneous running of eRHIC and RHIC, where a smaller amount of the ion beam-beam tune shift is allocated to the eRHIC collider, and dedicated eRHIC running where all available beam-beam tune shift is allocated to a single collision point. The luminosity as calculated by Eqn. (1) is several percent lower than the numbers reported in the ZDR for the ERL-ring cases and exactly the same as that reported in the ring-ring case. We believe the Eqn. (1) is reliable and the ZDR should be adjusted accordingly. Comparing the ERL-ring and ring-ring numbers for eRHIC, there is a factor of two luminosity increase from the increase of the ion current, allowed because the electrons may be more highly disrupted by the beam-beam force than is possible in rings. But the lion's share of the increase comes from the assumption that the beam-beam tune shift in the ion ring will be much higher. Perhaps surprisingly, none of the luminosity increase comes from assuming that the interaction point optics changes. Comparing ELIC and eRHIC numbers, it is clear that most of the advantage in the ELIC numbers comes from the smaller $\beta^*_{i,y}$. A smaller interaction point beta function is allowed because the higher repetition rate in the ELIC beams allows smaller charge-per-bunch for the same ion current, and it is therefore expected that longitudinal electron cooling is more highly effective in reducing ion bunch length for ELIC. The reduced ion bunch length allows the $\beta^*_{i,y}$ to be reduced to small values without the luminosity being reduced from the hourglass effect.

	ERL-ring	ring-ring
I_i (A)	0.9	0.45
ξ^i_y	0.007,0.024	0.0033
$\beta^*_{i,y}$ (cm)	26	27
σ_y / σ_x	1.0	0.5
L (standard units)	$2.6,9.0 \times 10^{33}$	4.5×10^{32}

Table 2: Comparison ERL-ring and ring-ring interaction point luminosities for eRHIC parameters

In this note an old formula is resurrected relating the ultimate luminosity in a collider to ion beam parameters only, under the assumption that the electron and ion beam spot sizes have the same *rms* values. This formula is used to reproduce the standard luminosity numbers for ELIC and eRHIC. The formula clearly shows that luminosity increases in going to ERL-ring colliders must be tied to: (1) an increase of the ion current, (2) an increase in the ion beam-beam tune shift, or (3) a decrease in the collision point beta function. In particular, the electron beam emittance does not appear in the formula, and seems not provide much leverage to increase the luminosity of ERL-ring colliders.

References

[1] Merminga, Douglas, and Krafft, Annual Reviews (2003)
[2] Appendix A: Complete ELIC Parameters: Draft ELIC ZDR
[3] Brookhaven ZDR