Field Error Tolerances for Spreader/Recombiner Magnets

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1. Field Error Tolerances – Analytic Approach

Analytic formalism describing Courant-Snyder invariant mismatch and the emittance dilution due to magnet field errors (represented by a specific multipole content) has been described in detail in a separate document: JLAB-TN-06-030. Here, we will use Eq. (10-17) from JLAB-TN-06-030 to analyze multipole tolerances for Spreader/Recombimer magnets for 12 GeV CEBAF.

To re-cap the analytic formulas, Eq. (10-17); the linear errors, m =1, cause the betatron mismatch – invariant ellipse distortion from the design ellipse without changing its area – no emittance increase.

$$\left(\frac{\sigma_\varepsilon}{\varepsilon}\right)_{mis} = \sqrt{\frac{1}{2}\sum_{n=1}^{N}\left(\beta_n \delta\phi_1\right)^2} = \sqrt{\frac{1}{2}\Delta\phi_1^2 \sum_{n=1}^{N}\left(\beta_n\right)_{quad}^2 + \frac{1}{2}\delta\phi_1^2 \sum_{n=1}^{N}\left(\beta_n\right)_{dipole}^2} \qquad (1)$$

The higher, m > 1, multipoles will contribute to the emittance dilution – 'limited' by design via a separate allowance per each segment (Spr/Rec). Here one assumes the following multipole content for the dipoles and quads:

- Quads: sextupole (m = 2), octupole (m = 3), duodecapole (m = 5) and isacopole (m = 9)
- Dipoles: sextupole (m = 2) and decapole (m = 4)

where

$$\left(\frac{\sigma_\varepsilon}{\varepsilon}\right)_{dil} = \sqrt{\frac{1}{2}\sum_{n=1}^{N}\left(\beta_n \delta\phi\right)^2} = \sqrt{\frac{1}{2}\Delta\phi_{quad}^2 \sum_{n=1}^{N}\left(\beta_n\right)_{quad}^2 + \frac{1}{2}\Delta\phi_{dipole}^2 \sum_{n=1}^{N}\left(\beta_n\right)_{dipole}^2} \qquad (2)$$

$$\Delta\phi_{quad} = a\sqrt{\frac{3}{2}\frac{1}{2^2}(2\times2)\delta\phi_2^{\,2} + \frac{5}{2}\frac{1}{2^4}X^{\,2}\left[(3\times3)\delta\phi_3^{\,2} + 2(1\times5)\delta\phi_1\delta\phi_5\right] + \frac{9}{2}\frac{1}{2^8}X^{\,6}\left[(5\times5)\delta\phi_5^{\,2} + 2(1\times9)\delta\phi_1\delta\phi_9\right]}$$

$$\Delta\phi_{dipole} = a\sqrt{\frac{3}{2}\frac{1}{2^2}(2\times2)\delta\phi_2^{\,2} + 2\frac{5}{2}\frac{1}{2^4}X^{\,2}(2\times4)\delta\phi_2\delta\phi_4 + ...}$$

Here
$$\phi_n = \frac{\int G_n dl}{B\rho} = \int k_n dl$$

is the integrated multipole moment in the geometric units, where (3)

$$G_m = \frac{1}{r_0^{\ m}} B_{m+1} \ \left[kGauss\ cm^{-m} \right] \qquad\qquad k_n = \frac{G_n}{B\rho} \ \left[cm^{-(n+1)} \right]$$

where, one uses multipole expansion coefficients of the azimuthal magnetic field, B_θ, given by the standard Fourier series representation in polar coordinates (at a given point along the trajectory):

$$B_\theta(r,\theta) = \sum_{m=2} \left(\frac{r}{r_0} \right)^{m\text{-}1} \left(B_m \cos m\theta + A_m \sin m\theta \right) \tag{4}$$

The analytic formalism from JLAB-TN-06-030 allows for the beam trajectory to be perturbed with respect to the design orbit, so that the beam centroid undergoes a betatron oscillation with amplitude X_0. The intrinsic beam size in Eq. (2) is defined as a = 4σ. For details, see Figure 2 in JLAB-TN-06-030).

2. Magnet Tolerances for 12 GeV CEBAF Lattice

The limits on ‘tolerable’ magnet errors will be set in terms of specific allowances for betatron mismatch (10% increase) and emittance dilution (10% increase) through each beamline (Spr/Rec).

The linear errors, focusing (m =1), cause the betatron mismatch (invariant ellipse distortion from the design) without emittance increase. The sources of the betatron mismatch come from the quad gradient errors and the dipole body gradients (to be compensated by the dedicated matching quads)

The higher, m > 1, multipoles will contribute to the emittance dilution – ‘limited’ by design via a separate allowance.

Using Spr/Rec 1-10 optics one can evaluate Eq.(1) and (2) for each segment. Table 1 (quads) and Table 2 (dipoles) summarize focusing error tolerances (10% mismatch) and field quality specs – higher multipoles (10% emittance dilution) for groups of quads and dipoles in the corresponding Spr/Rec sections. The values of multipoles are calculated in the extreme case – a given order (m) multipole by itself exhausts the emittance dilution allowance of 10%.

section	quad type	$\sqrt{\frac{1}{2}\sum_{n=1}^{N}\beta_n^2}$ $[cm]$	$\Delta\phi_1\left[cm^{-1}\right]$	$\phi_1^{max}\left[cm^{-1}\right]$	$\Delta\phi_1/\phi_1^{max}$	ε_x $[cm\ rad]$	$\langle\beta_x\rangle$ $[cm]$	σ_x $[cm]$	$\Delta\phi_2$ $[cm^{-2}]$	$\Delta\phi_3$ $[cm^{-3}]$	$\Delta\phi_5$ $[cm^{-5}]$	$\Delta\phi_9$ $[cm^{-9}]$
Spr/Rec 1	MQB	10965	9.12E-06	4.41E-03	2.07E-03	4.0E-08	2360	9.7E-03	8.5E-05	1.3E-04	5.0E-04	1.5E-02
Spr/Rec 2	MQC	24773	4.04E-06	7.47E-03	5.41E-04	3.0E-08	4730	1.2E-02	3.0E-05	4.4E-05	1.6E-04	4.5E-03
Spr/Rec 3	MQA	20257	4.94E-06	4.94E-03	1.00E-03	2.0E-08	4194	9.2E-03	4.9E-05	7.5E-05	3.0E-04	9.3E-03
Spr/Rec 4	MQA	12599	7.94E-06	5.77E-03	1.37E-03	2.0E-08	2828	7.5E-03	9.8E-05	1.5E-04	6.3E-04	2.1E-02
Spr/Rec 5	MQA	13661	7.32E-06	5.47E-03	1.34E-03	3.0E-08	2840	9.2E-03	7.2E-05	1.1E-04	4.4E-04	1.4E-02
Spr/Rec 6	MQA	14853	6.73E-06	5.89E-03	1.14E-03	7.0E-08	6868	2.2E-02	2.4E-05	3.2E-05	9.6E-05	1.7E-03
Spr/Rec 7	MQA	26139	3.83E-06	5.59E-03	6.85E-04	1.2E-07	5379	2.5E-02	1.1E-05	1.5E-05	4.1E-05	6.4E-04
Spr/Rec 8	MQA	36216	2.76E-06	4.66E-03	5.92E-04	1.9E-07	6565	3.5E-02	5.4E-06	6.3E-06	1.5E-05	1.6E-04
Spr/Rec 9	MQA	30293	3.30E-06	2.06E-03	1.61E-03	3.6E-07	6229	4.7E-02	4.4E-06	4.6E-06	8.6E-06	6.1E-05
Spr/Rec A	MQA	26485	3.78E-06	4.56E-03	8.28E-04	5.0E-07	5052	5.0E-02	4.6E-06	4.7E-06	8.4E-06	5.4E-05

Table 1 Quadrupole magnet specs (integrated gradient, sextupole, octupole, duodecapole and isacopole errors)

section	$\sqrt{\frac{1}{2}\sum_{n=1}^{N}\beta_n^2}$ $[cm]$	$\delta\phi_1\left[cm^{-1}\right]$	$\delta\phi_1/\phi_1^{max}$	ε_x $[cm\ rad]$	$\langle\beta_x\rangle$ $[cm]$	σ_x $[cm]$	$\Delta\phi_2$ $[cm^{-2}]$	$\Delta\phi_4$ $[cm^{-4}]$
Spr/Rec 1	3739	2.67E-05	6.07E-03	4.00E-08	1396	0.007	3.32E-04	9.99E-04
Spr/Rec 2	8269	1.21E-05	1.62E-03	3.00E-08	831	0.005	2.32E-04	7.41E-04
Spr/Rec 3	8872	1.13E-05	2.28E-03	2.00E-08	2975	0.008	1.35E-04	4.04E-04
Spr/Rec 4	14561	6.87E-06	1.19E-03	2.00E-08	3620	0.009	7.40E-05	2.17E-04
Spr/Rec 5	9892	1.01E-05	1.85E-03	3.00E-08	3655	0.010	8.65E-05	2.42E-04
Spr/Rec 6	18525	5.40E-06	9.17E-04	7.00E-08	6135	0.021	2.08E-05	4.65E-05
Spr/Rec 7	20175	4.96E-06	8.87E-04	1.20E-07	7274	0.030	1.23E-05	2.30E-05
Spr/Rec 8	25195	3.97E-06	8.51E-04	1.90E-07	8436	0.040	6.60E-06	1.02E-05
Spr/Rec 9	36192	2.76E-06	1.34E-03	3.60E-07	13358	0.069	2.11E-06	2.07E-06
Spr/Rec A	35745	2.80E-06	6.14E-04	5.00E-07	13847	0.083	1.63E-06	1.33E-06

Table 2 Dipole magnet specs (integrated quadruople body gradient, sextupole and decapole)