Calculating Multipoles in Opera/TOSCA - mesh and software noise effects

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Introduction

Multipoles must be assigned to dipoles and quadrupoles to model an accelerator. Multipoles were measured for quadrupoles during initial construction.(1). Dipole multipoles were not measured per se, instead the departure from linearity across the magnet midplane was measured and compared to a spec derived from focusing effects. (2) Vector Fields Opera post-processor has a pair of commands allowing one to calculate multipoles to nth order on a specified circle. Thus detailed dipole models may be used to provide input to accelerator models. This paper explores the detail needed. Mechanically perfect magnets are assumed.

Two magnets were examined, the S/R dipole which has 10cm pole and is 98cm long; and the proposed 4m arc dipole for arc 10. The former is a modified C and the latter a symmetric H. Both were modeled without midplane symmetry so saturation effects in the pole could generate skew as well as normal terms. The skew terms proved very useful in finding limits in the software. The former magnet is well into saturation in 6R, the example used, and the latter just entering saturation. The models are larger than any previously known to Vector Fields support.

Beam orbits were approximated with arcs of 1 cm radius circles properly sized and located within the magnet and extending 25cm beyond it. Circles were spaced every 2.5mm. The difference between such arcs and the path actually taken by the beam outside the steel is of order 1mm at 25cm from the steel. Since the field is at most 10G there, the multipole contribution of the difference is not significant. All magnets are modeled as horizontal bends with the center of the pole as (0,0,0).

Results on one meter magnet

The upper plot on the next page shows the first four normal multipoles in a model of the H-modified AB magnet to be used in 6R. It runs at 14 kG in the baseline. The lower plot shows the same data at different scale so one can see these components within the body of the dipole. On the second page following the skew terms are plotted. The large amplitude normal components are due to pole edge saturation. Half-sagitta is only 0.4 cm; pole half-width 5.1 cm. At 12 kG the multipoles are a factor of five lower. Cos1 is the quadrupole, cos2 the sextupole, cos3 the octupole, cos4 the decapole. Cos2 and cos4 are allowed by geometry.

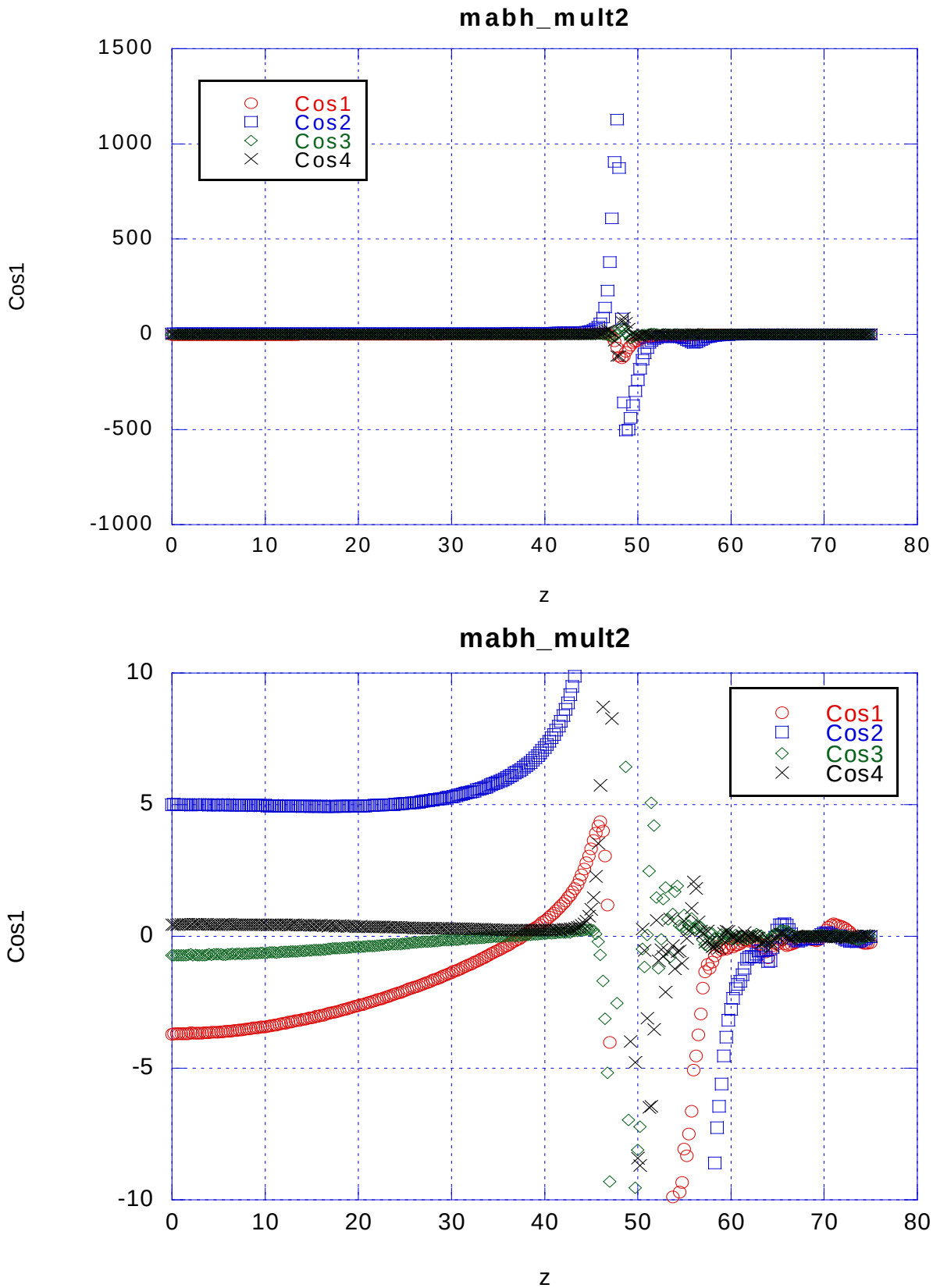

Figure 1. Normal multipoles (G) of MABH6R dipole.

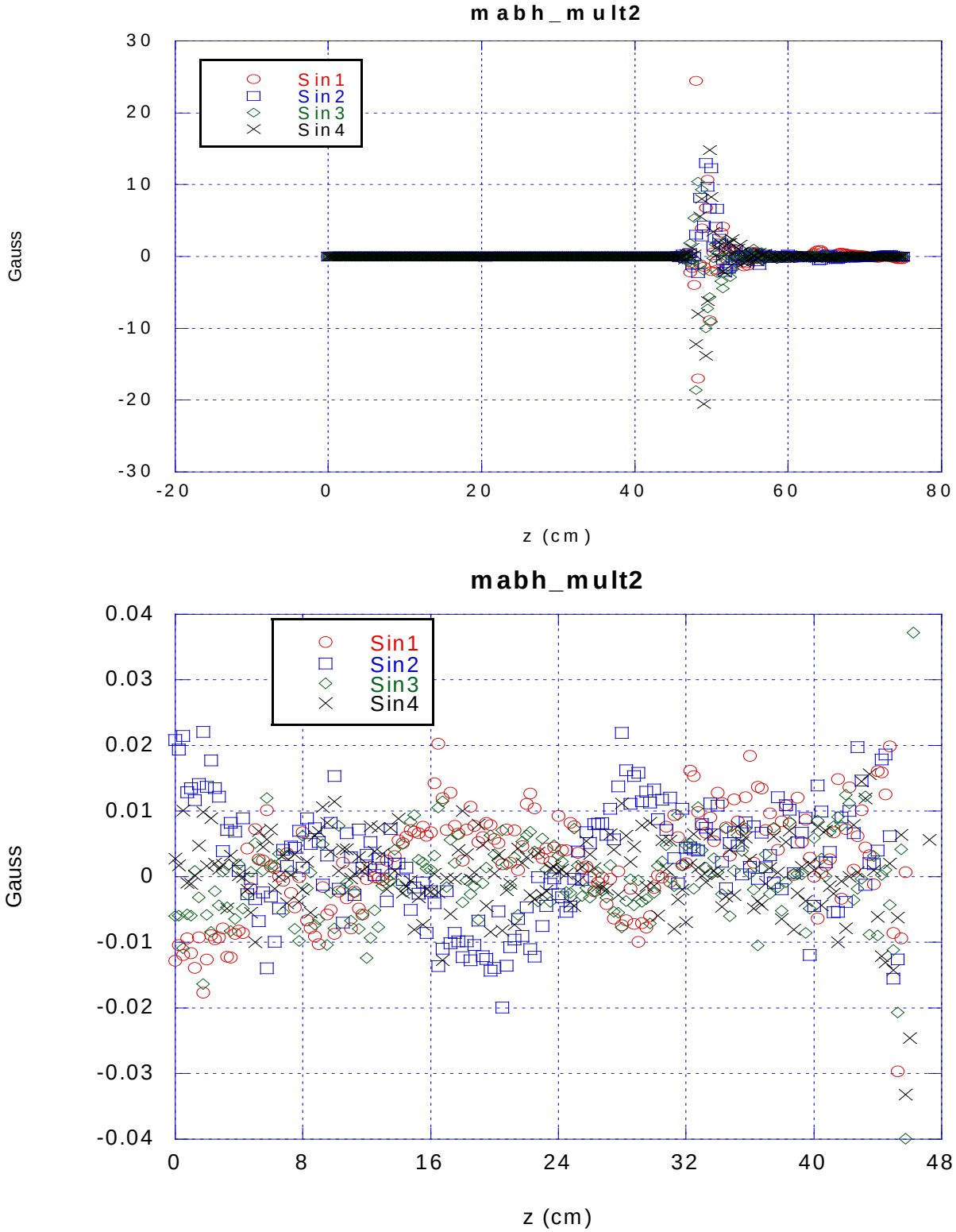

Figure 2. Skew multipoles of MABH6R. The oscillations in the lower graph are what set me off on an exploration of mesh and model background effects on multipoles. There is no physical basis for them.

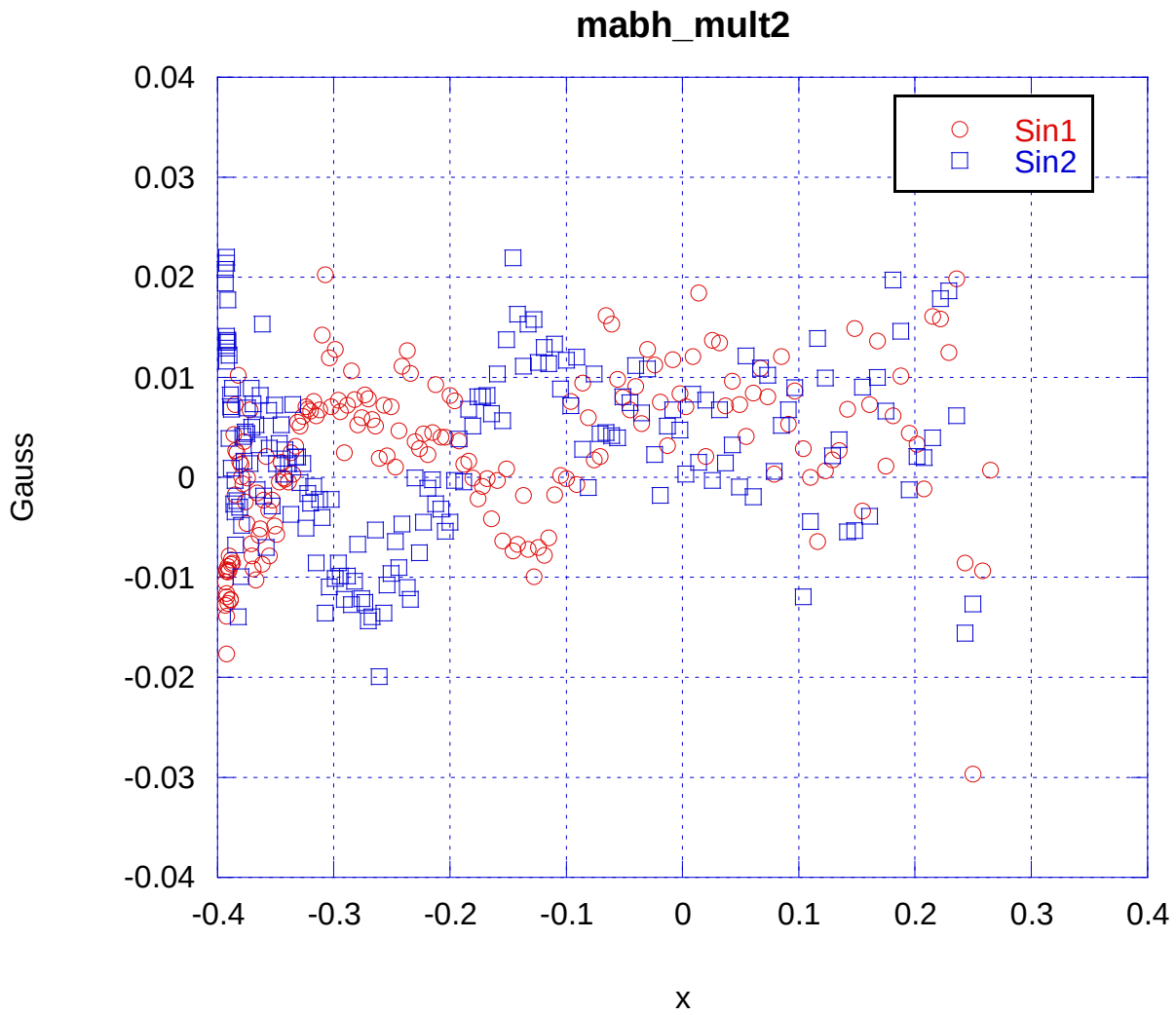

Figure 3. First two skew terms plotted against transverse position. Beam enters the steel at x=0.393cm and in the midplane is at x= -0.393cm. Since the beam is at an angle of 1.7 degrees at the entrance, the circle on which the multipoles are calculated has substantial gradients across it, causing the large values near Z=48 and x=0.4.

One of the display options in the post-processor is "view cartesian patch". The field is calculated on a specified plane at constant value of the third coordinate. On the next few pages are examples.

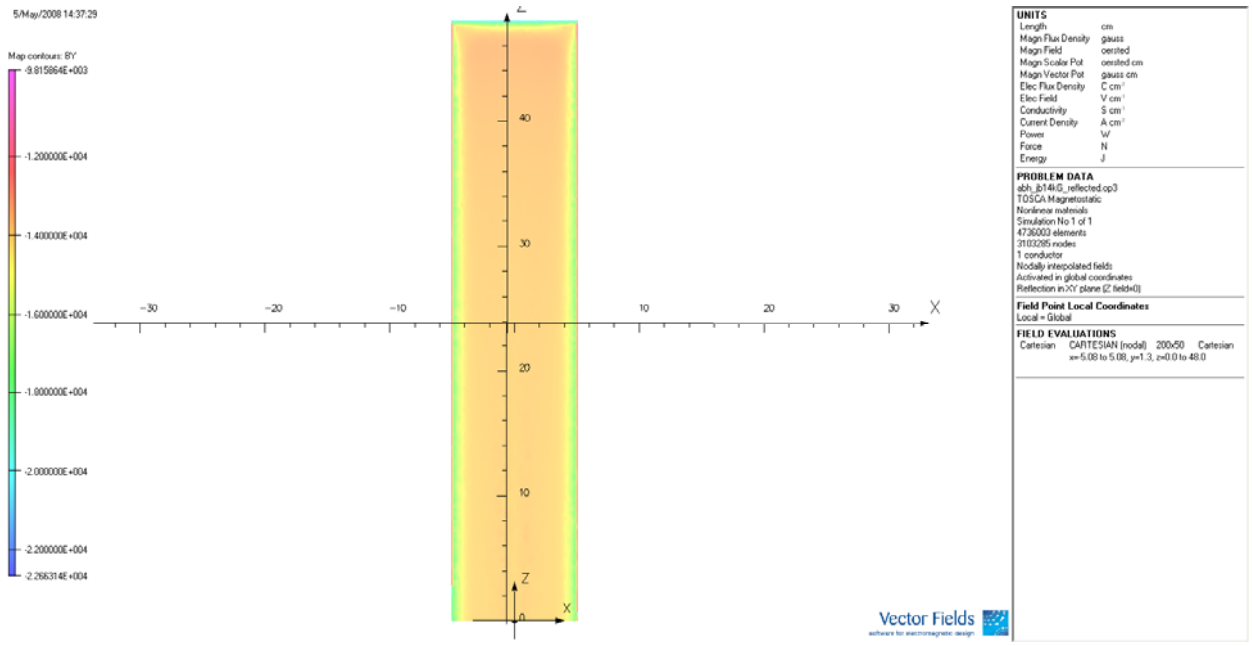

Figure 4a. Cut through MABH6R pole at y=1.3cm, 50 microns inside the steel

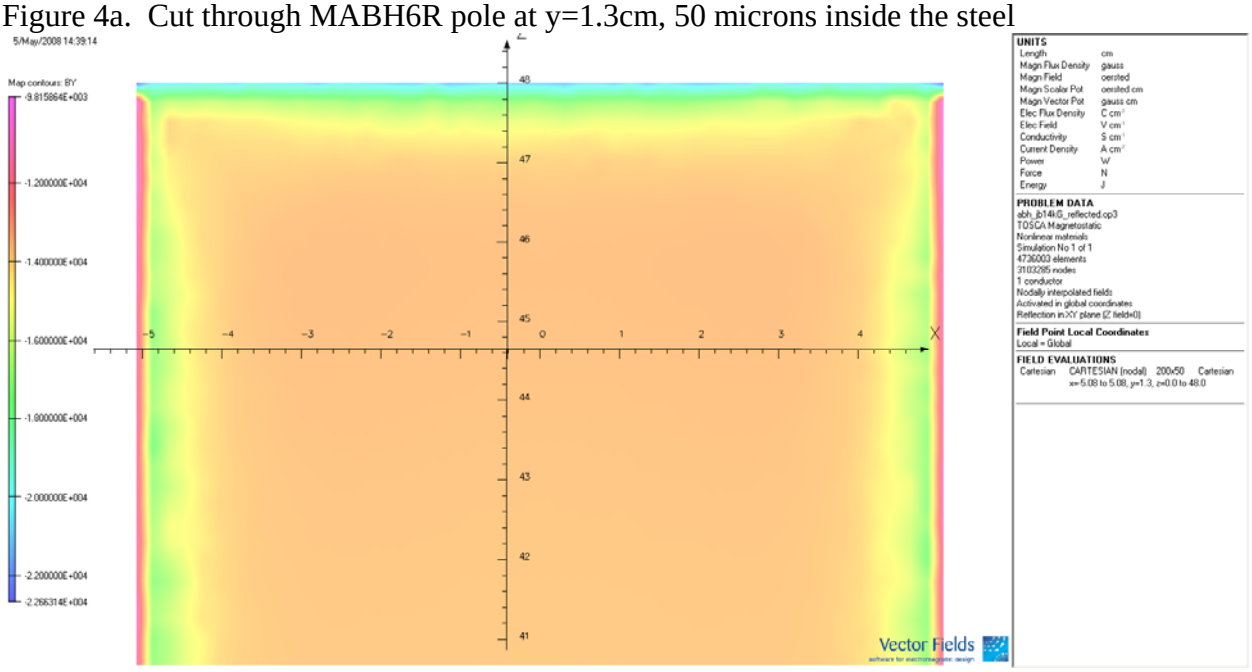

Figure 4b. Same as above, magnified so saturation at the end of the pole may be seen.

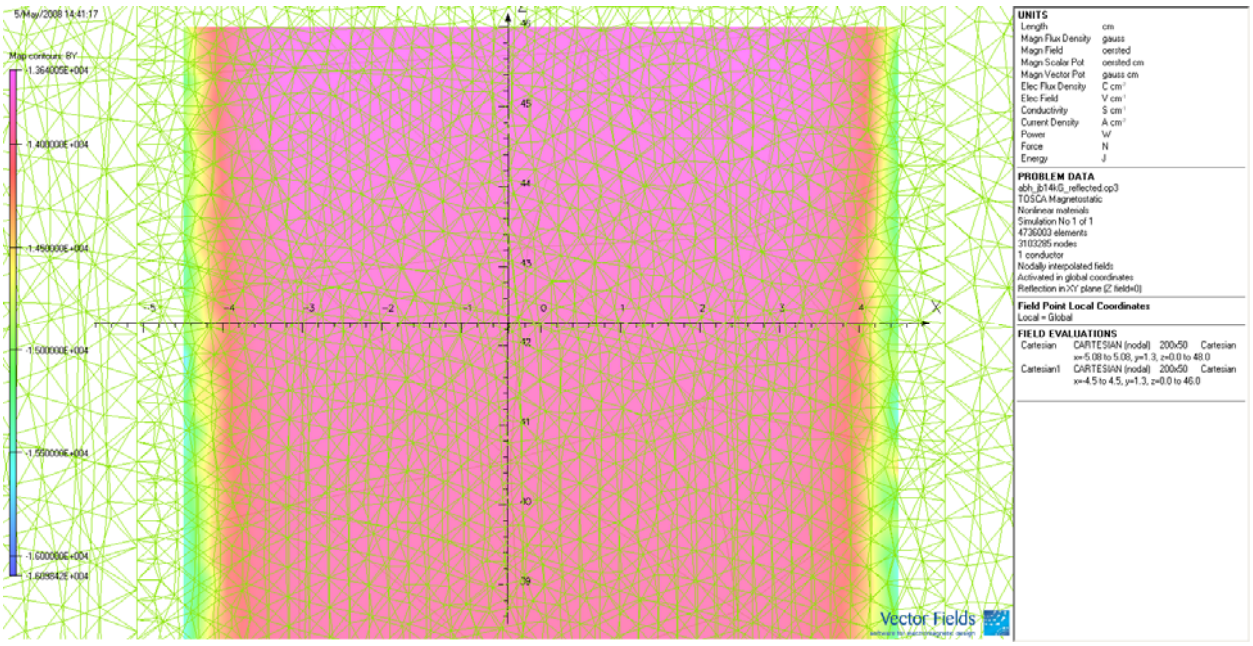

Figure 4c. Cartesian plot evaluated to Z=46cm, not 48cm and x=[-4.5,4.5] instead of full pole width to limit field range to ~2500G. Mesh display turned on. Nominal mesh size 2.5mm. Notice the mottling along the edges of the pole.

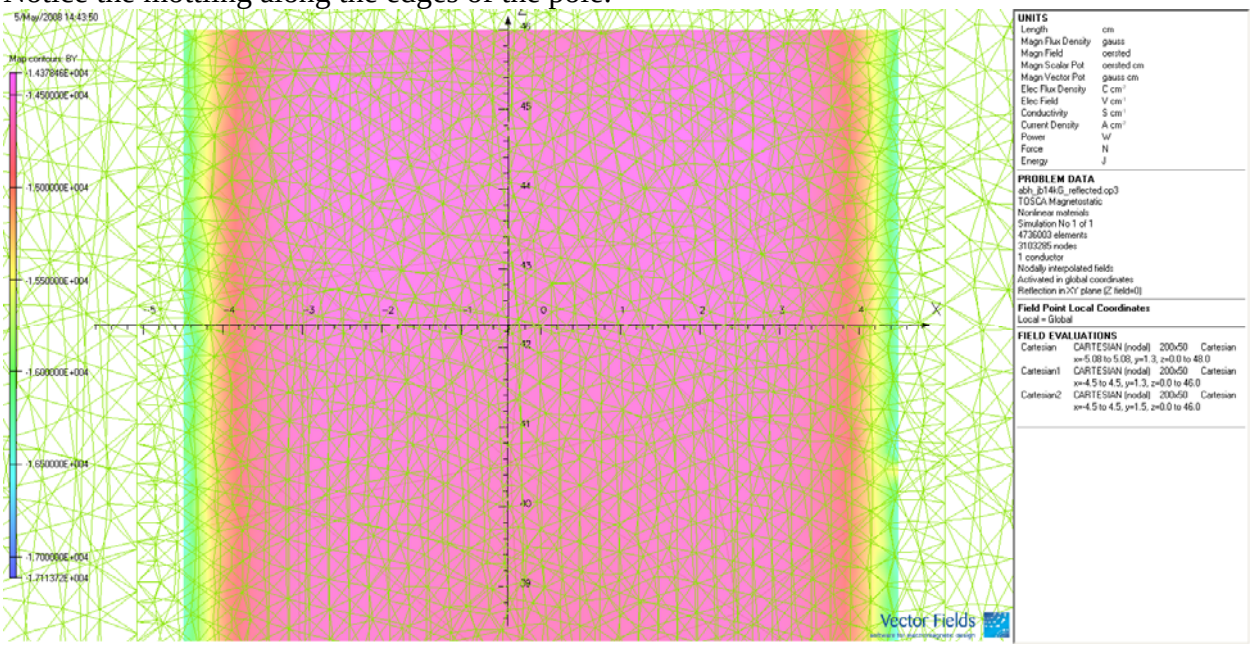

Figure 4d, same as 4c except at y=1.5cm,2mm into the pole. Mottling is reduced, or at least I can convince myself of that.

I concluded by looking at such plots on three models of this magnet that mesh size and the periodicity induced by the mesh generator could help create the non-physical variations in skew terms seen in figures 2 and 3. Background size, aka boundary location where tangential fields are set to zero, might also have an effect, as in (3). The method of field calculation might also play a role. Since the integration method takes two orders of magnitude longer to complete than nodal interpolation, I decided to first investigate the effect of the number of points at which the

field is calculated on the 1cm radius circle on the multipole amplitude. I therefore ran the post-processor asking for multipoles on circles with 25, 50, 60, 75, 100 and 120 points. Recall that the mesh size is 2.5mm so a 1cm radius circle will have about 25 nodes. For icosapole, the sampling theorem sets the minimum at 20. I used a quadratic mesh, which doubles the effective number. For the large value normal terms the number of points doesn't matter (figure 5a). For the skew terms it does (figure 5b). Thereafter I used 60 points on my circles.

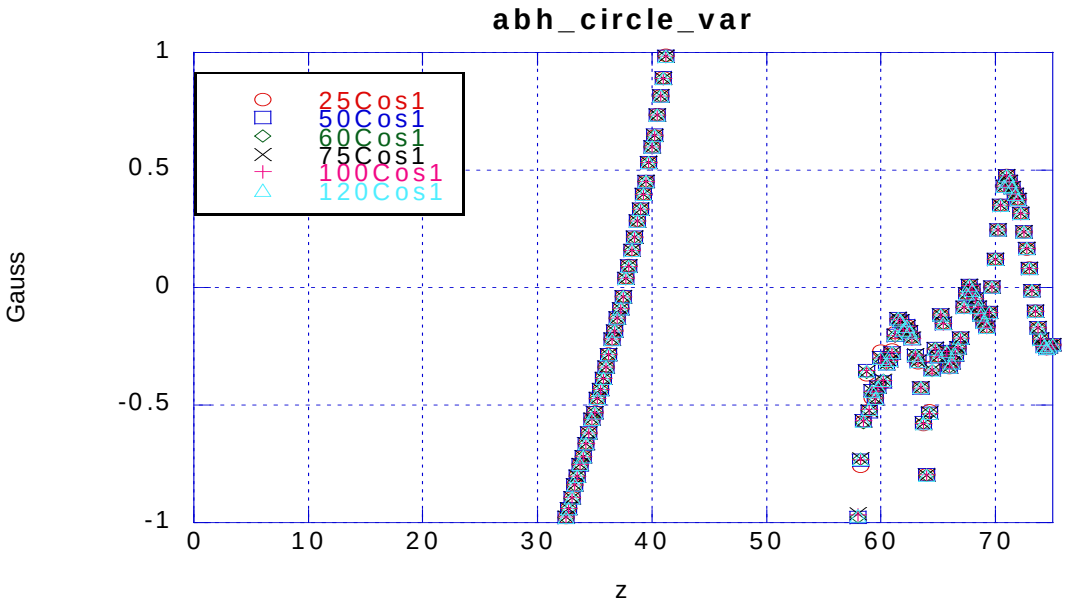

Figure 5a Normal quadrupole as a function of number of points on circles

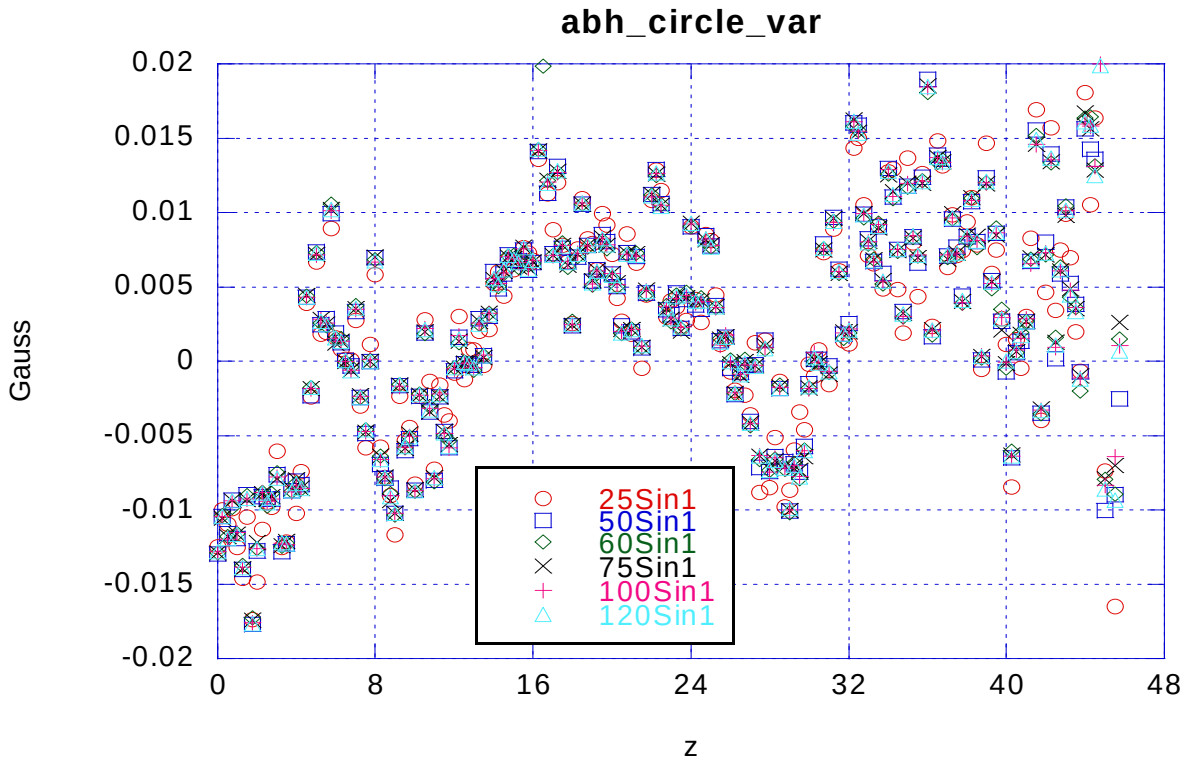

Figure 5b. Skew quadrupole as a function of number of points. Notice that the red circles (25 pts) and blue squares (50 pts) are not always superimposed on the four other symbols.

Results with 4m arc dipole (MXP)

The questions raised by the 1m 6R dipole were investigated not with it but with the 4m arc dipole as I was asked to turn my attention to that magnet. One fourth of this H magnet was modeled, assuming symmetry around the Z=0 and X=0 planes. Symmetry around Y plane was not assumed since I wanted to see the skew terms. Since this model has four times the steel length the models are much larger than those of the 1m dipole. This increases calculation time. It also confuses the mesh generator, so planes were inserted every 20/25 cm in the model to "re-orient" the mesh generator. These have no physical meaning. See also (3).

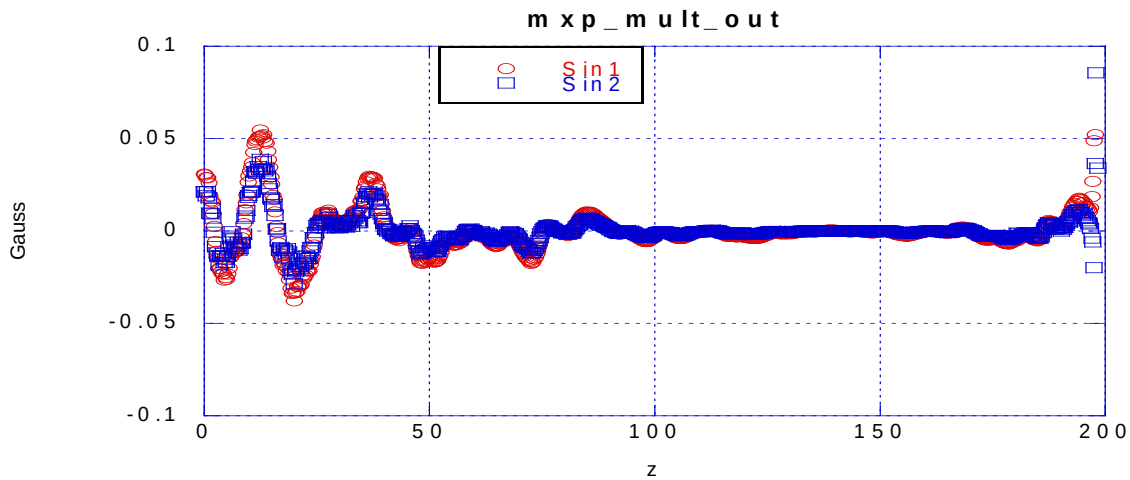

Figure 6a. First two skew terms in 4m arc dipole. 2.5 mm mesh, large background volume.

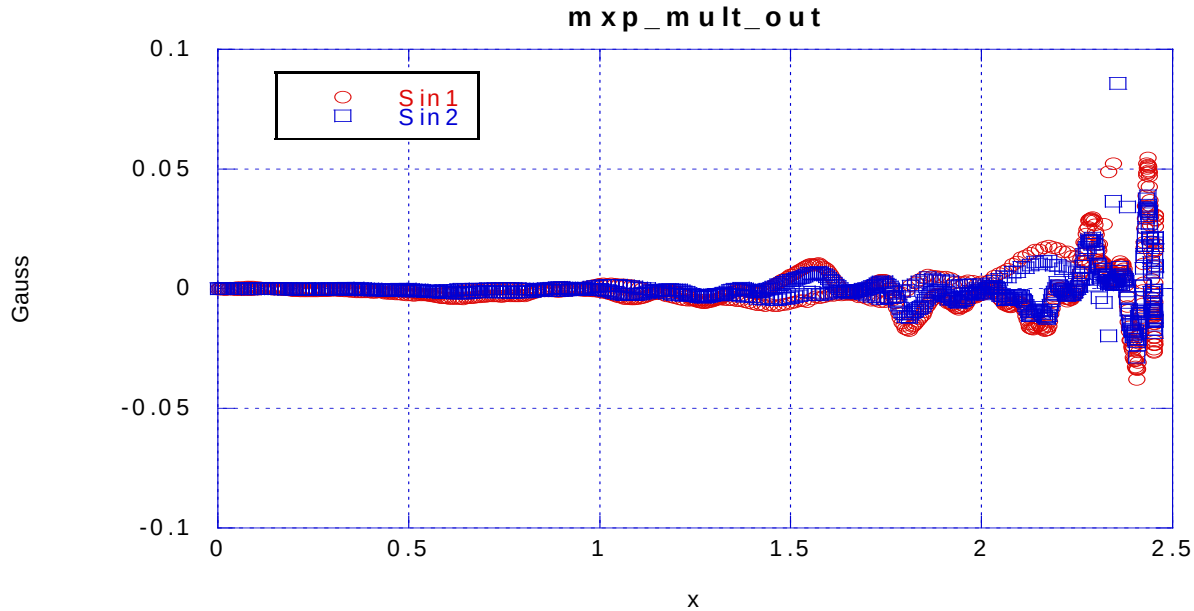

Figure 6b. As above, plotted against x within steel. Half-sagitta 2.55 cm. Pole half-width 6.51 cm. The phenomenon occurs when the beam is close to an edge of the pole. Recall the mottling in figures 4c and 4d. There are two Y values for each X because of the symmetry assumptions in the model - I have to reflect the beam (x and angle) from the -x to the +x side of the magnet because calculation can't occur outside the model proper.

The half-sagitta in the arc dipoles is ~40% of the MXP dipole pole half-width versus 8% in the MABH. The mottling seen in figure 4c and 4d is present in the model used for figure 6, albeit not as pronounced. Output file size 2 GB.

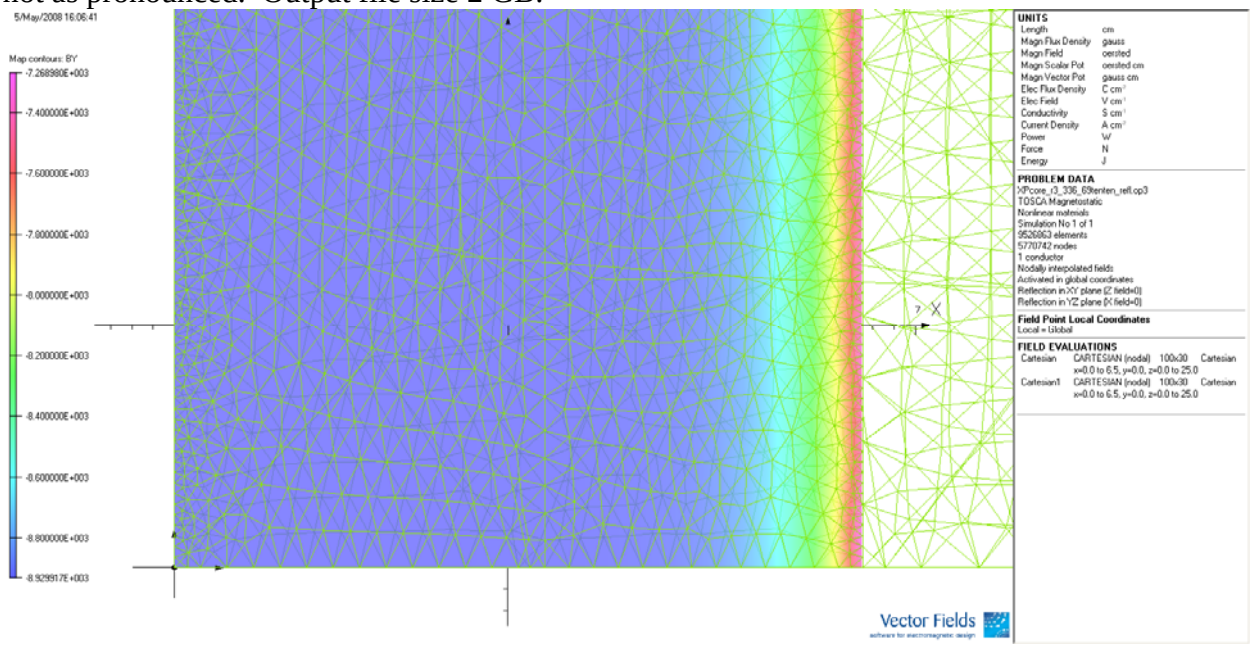

Figure 7. Cartesian patch at y=1.3cm near middle of XP pole. 2.5mm mesh.

I decided to decrease the mesh size to 2mm in the first cm of pole steel and in the gap air. The output file for this model is over 4GB; solution took just over four days.

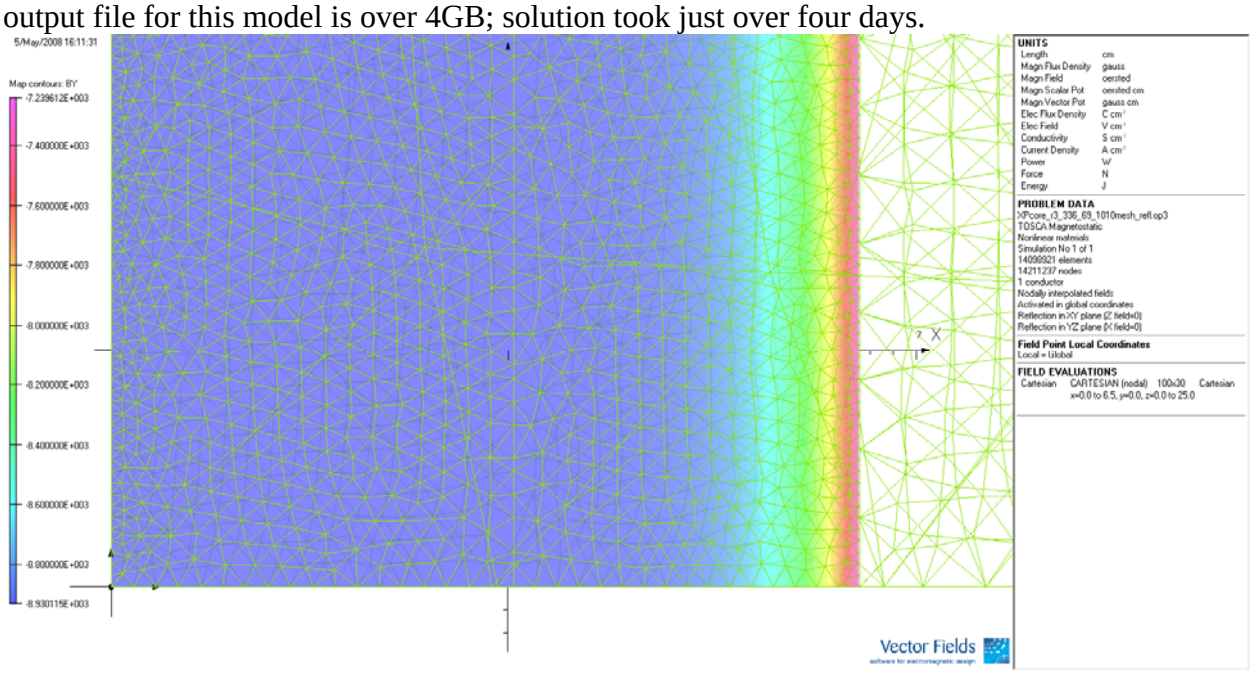

Figure 8. Cartesian patch with 2mm mesh. Are the color bands on the right a bit smoother here than in figure 7?

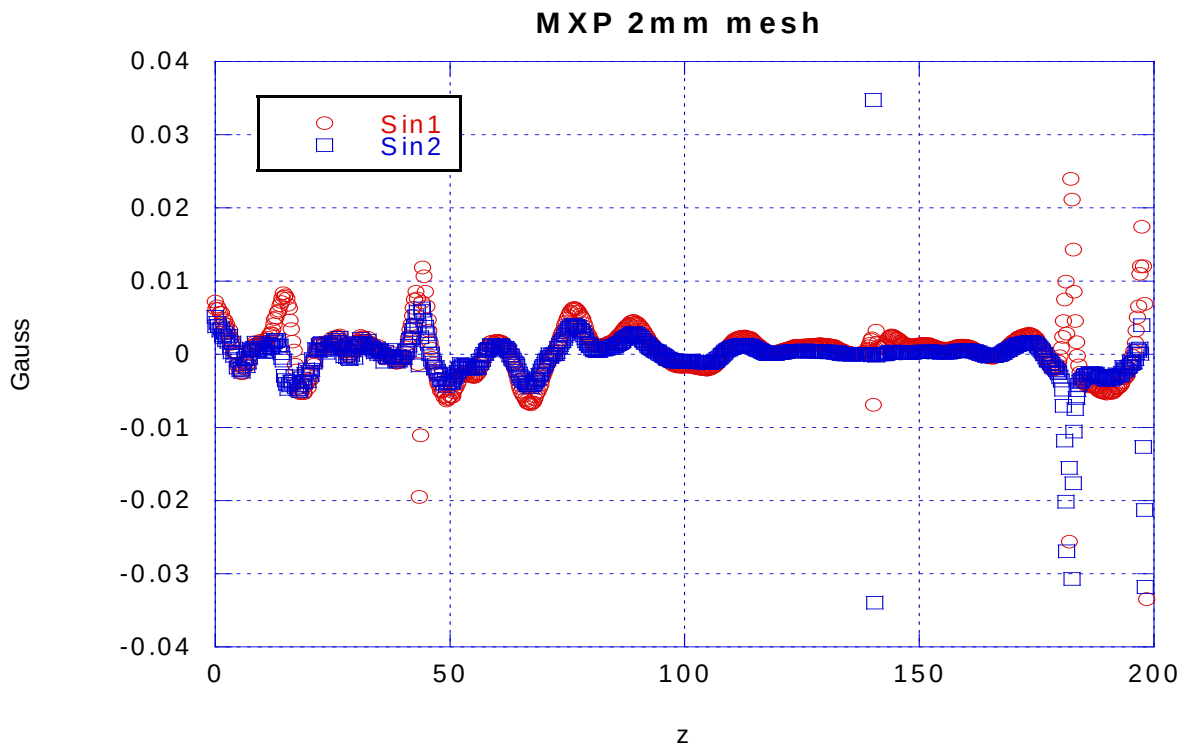

Figure 9. First two skew terms versus Z for 2mm mesh model. Amplitudes are a fifth of those in figure 6. The anomaly on the right is at and just after the cutting plane at 175cm. The mesh generator did not do a smooth job there.

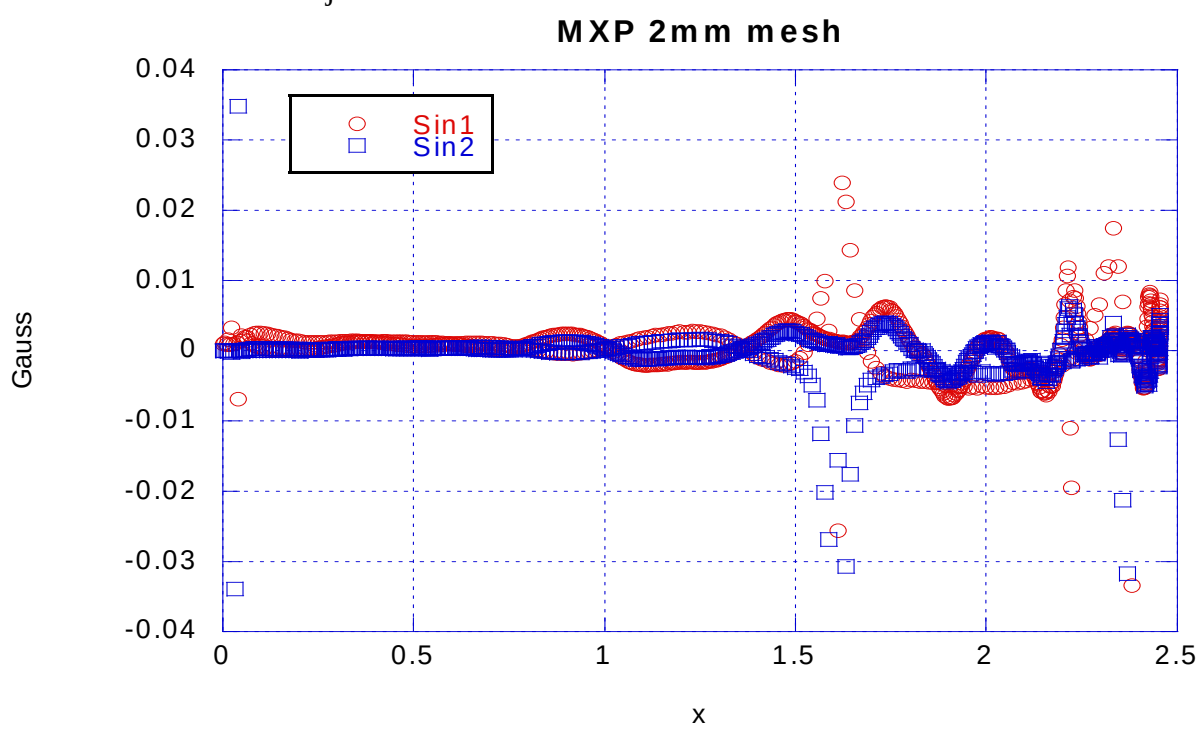

Figure 10. X dependence of 2mm mesh model. The cutting plane is the anomaly around x=1.7 cm.

Clearly a 2mm mesh is better than a 2.5mm mesh. The former model has 14.1 million elements, the latter 9.5 million. I created a model with mesh size of 1mm on the pole faces. This had 26 million elements and was too large for solution with my 16 GB RAM PC if I wanted to do anything else. I have solved two 19 million element models of the XP simultaneously in 4.5 days - doing nothing else. Typical page fault count in Task Manager: 11 GB.

I also ran a case, since deleted, in which the outer half of the air in the pole gap was meshed at 3.3 mm and the inner half, where the beam is, at 2.5mm. This was twice as bad as figure 6. This was likely due to the fact that the pole face mesh size has to match the gap mesh size in the first element, increasing "mottling" at the edge.

At the bottom of page 6 I wrote of the other method of field calculation, integration. I post-processed the 2.5mm mesh model on the same 800 circles graphed in figure 6 with integration instead of nodal interpolation. This took 5.5 days to complete. Results are non-physical and therefore worthless. There is no way that a perfectly symmetric dipole of this size can have gauss of skew field. In (3) I concluded that the difference in the vicinity of the beam for nodal interpolation versus integration could be mitigated if the background was big enough. Here I conclude that integration mode is totally worthless except for low field regions near the boundary of the model, regions irrelevant to accelerator modeling.

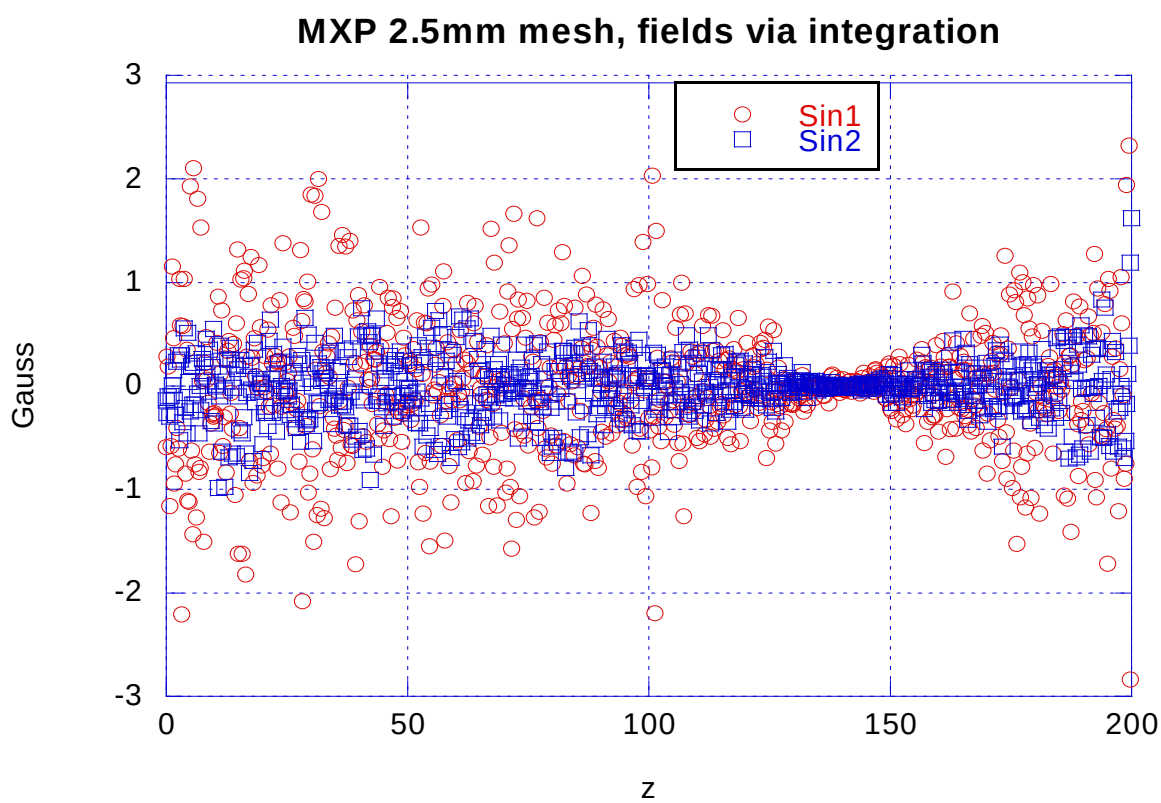

Figure 11. Same model as figure 6a. Evaluation using integration instead of nodal interpolation. Clearly non-physical. The neck around 140 cm is where the beam is close to x=0.

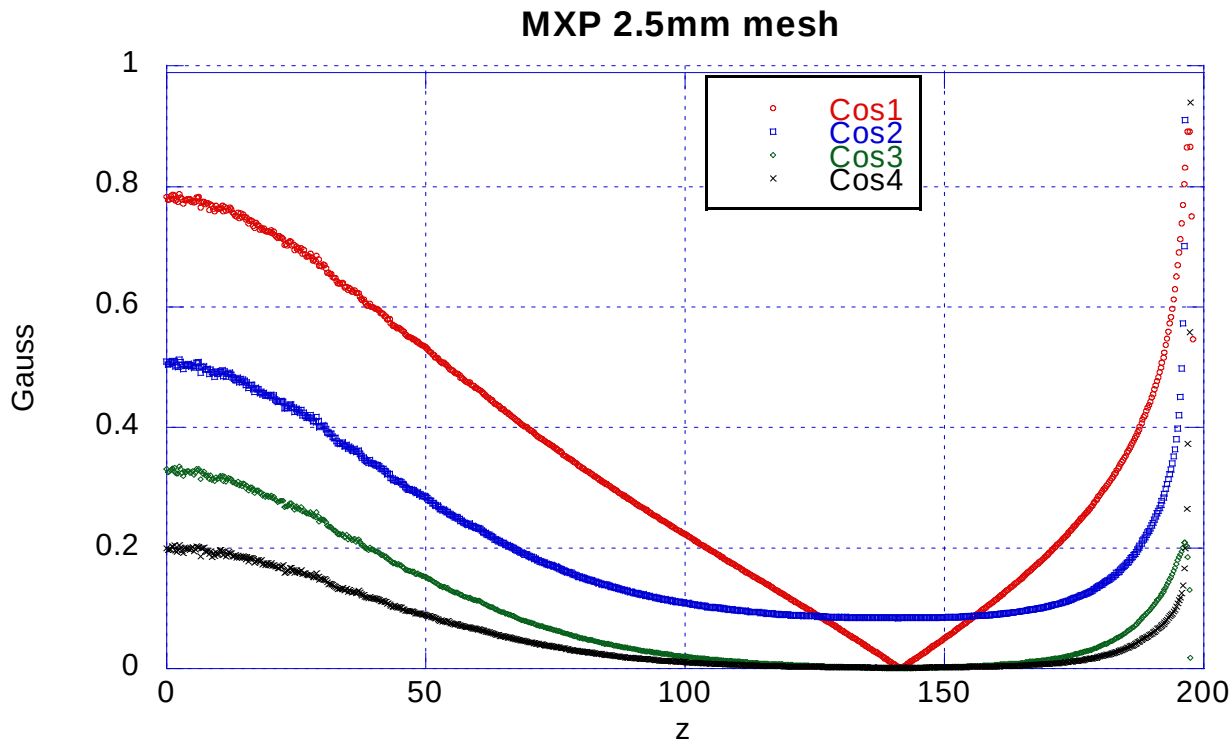

Figure 12. For completeness, normal multipoles in model of figure 6. One can see a bit of noise at low Z, perhaps at the +-10mG level. Noise is smaller than in figure 6a, the skews. Compare amplitudes with figure 1 (bottom) to get some feel for saturation effects.

Conclusions

1. Forget about the integration option for field evaluation. It's never worth the time on problems relevant to accelerators.
2. With a 2.5mm mesh, multipole values under 50 mG are numerical noise. With 2mm mesh, 10 mG. The bending field of the MXP is 9 kG, so the former is ~6ppm of the bending field. Normal multipoles are fractional gauss (figure 12) so a 2mm mesh has S:N of at least 20:1 through decapole. Given manufacturing tolerances, 2.5mm suffices for practical purposes.
3. Manufacturing tolerance studies must have large enough perturbations that the values cited in conclusion two are a modest fraction of the answer. Values at small perturbation must be extrapolated from the large perturbation results rather than calculated directly.
4. From reference (3) - Opera/TOSCA, when used properly, can produce multipole values which are useful for accelerator modeling.

References

1. QA Quadrupole Field Quality, Jay Benesch, TN-07-006
2. Dipole Field Inhomogeneities and the "Yunn Effect", D. Douglas and B. Yunn, TN-97-019
3. The 3m arc dipole (BA) - modeling and comparison with sparse experimental data, Jay Benesch, TN-08-xxx