

Hall D optics options Jay Benesch and Yves Roblin

I. Introduction and background

Since receiving an informal communication 7/12/07 from 12 GeV Civil Engineering concerning the May 15, 2007 change notice altering the ramp angle to approximately eight degrees Jay has been working intermittently on the hall D optics. More than a dozen variations have been tried. Some of these are shown in sections III-VI. Jay matched each of ten input cases which span the **best quartile** of matching achieved in CEBAF (TN-05-074). Nominal (CD-2) 12 GeV input emittances and energy spread were used to compute beam sizes unless otherwise noted.

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A solution with four QC quads, nine QP quads and twelve QW quads was originally arrived at. This solution has three fewer girders than the CDR solution so there should be cost savings in spite of the need for a new quad type, the QW, two-thirds longer than the QP with the same cross-section. Later in the note this QW requirement is eliminated.

Because there is no way to null dispersion in the line until the top of the ramp, at least without superconducting quads in a location with no 4K helium, the transverse dimensions of the optics and dispersion are coupled. Numerical tuning was generally done using the first eight quads to constrain the six parameters at the start of the ramp and the maximum betas within the full line. Two or three optical elements in the ramp and the final quadruplet were then used to null dispersion and its derivative and get acceptable beam profile at the collimator. In one case the BT triplet had to be invoked as well to keep horizontal beam size acceptable in the ramp dipoles. 20A power supplies were assumed throughout and provide adequate headroom if input conditions are not too far outside the parameter range studied. Reduction in input beta functions and emittance growth via proposed but un-implemented main machine change suggestions will increase headroom. CASA has concurred in this change in tuning concept, modulo the next paragraph.

In response to a comment by Dave Douglas, Jay prepared two files with more even quadrupole layouts. Not perfectly even due to interferences with lower pass doglegs and arc dipoles, but close. Y. Roblin examined the possibilities provided by these to ease the matching task in operations in spite of the pervasive dispersion. Solutions with these layouts via blind application of Optim are shown on page 6. More systematic solutions by Roblin are on pages 7-9.

Why would one want another layout? Neither Jay in Optim nor Yves in *elegant* have been able to devise an objective function which will allow automated matching of all nine input parameter test cases for the nominal optics. Two or three require manual intervention. If an alternate layout allows a single objective function definition for all cases, it is likely a better solution for Operations. On the other hand, hall D will operate only with the machine at or close to full energy. The accelerator will likely be set up at this energy at most once a year. Using a layout which requires expert manual intervention to set up beam to hall D once a year may be much more cost effective than the alternate. This can't be determined until the alternate is known. There are also T_{ij6} effects in the present lattice which might confuse experts during setup.

II. Input from GlueX collaboration

Questions for Glue-X collaboration

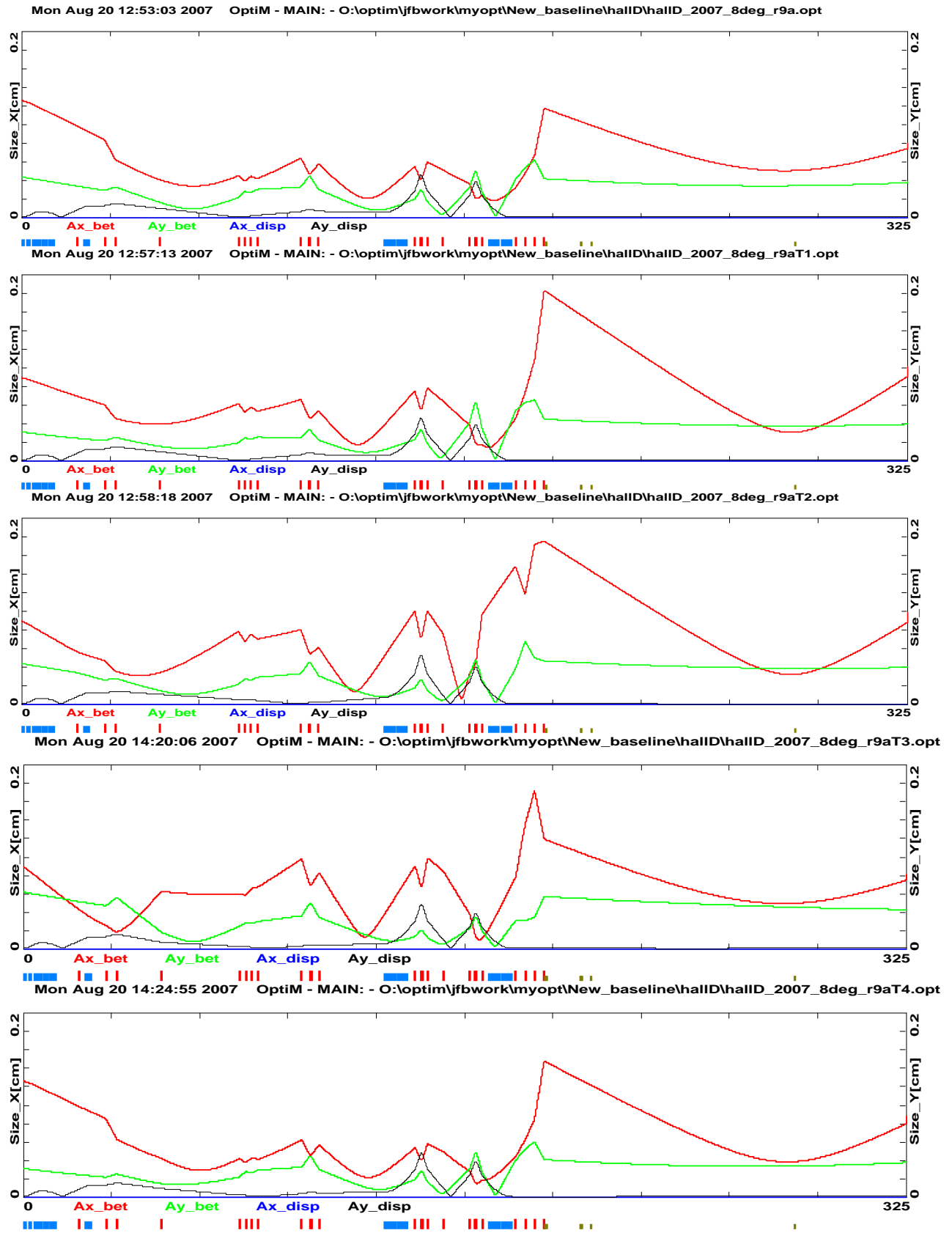
1. What is the maximum allowable current per unit area for the diamond? A 3 D plot or function would be nice: allowable coulombs versus sigmaX and sigmaY, Gaussian beam. SigmaX in range 0.4-1.6 mm, sigmaY range 0.2-0.8 mm.
2. What's the expected coulomb profile over the first five years? Diamond change after three years? Have you thought about the tradeoff of halo versus diamond life resulting from smaller spots?
3. If lattice changes cut the incoming betas and emittances each by a factor of two, can the spot size on the diamond be cut a factor two or will it have to be blown up to keep diamond damage down?

Answers from collaboration:

1. Studies are underway.
2. Will be informed by studies.
3. Spot size on diamond may be reduced. Diamond radiator will simply be moved so beam impinges on a new spot when accumulated damage reduces polarization.

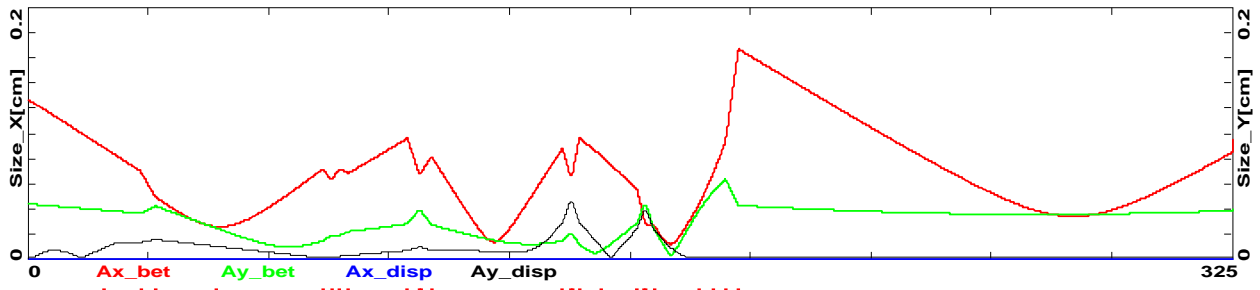
III. Jay's initial solution

The plots on the following two pages show the envelopes for the ten input variations in Jay's initial solution. The table following the plots gives the quad values for each.

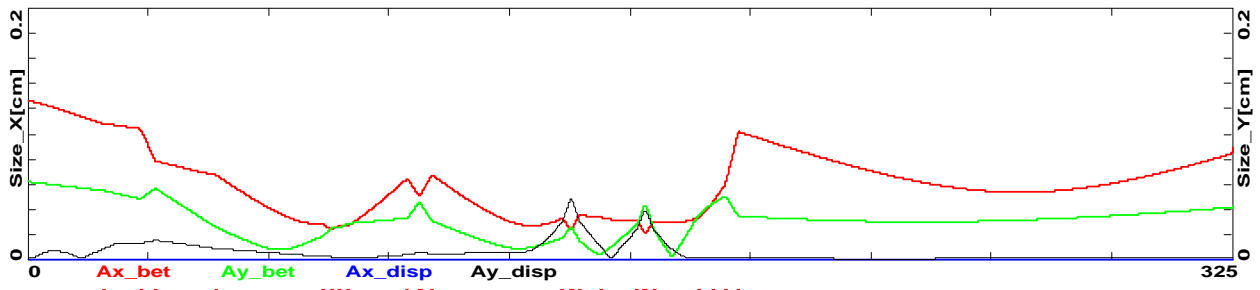


Beam sizes for nominal and four variations of input parameters to D line. Add black and green lines in quadrature to get vertical beam sigma. Red is horizontal. 2mm full scale.

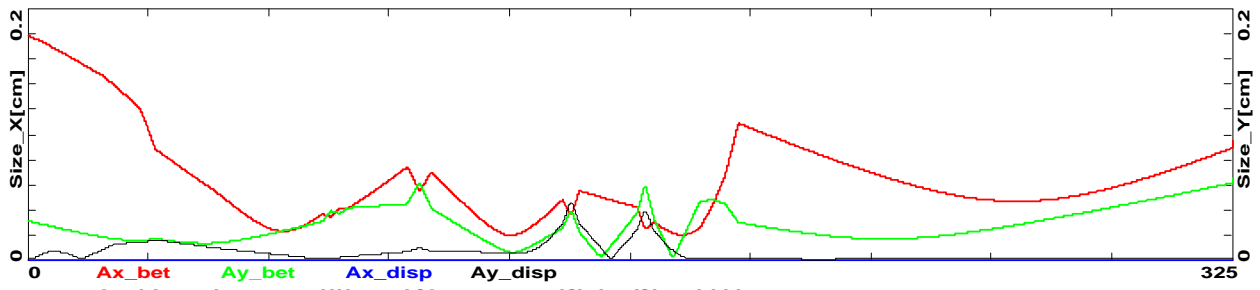
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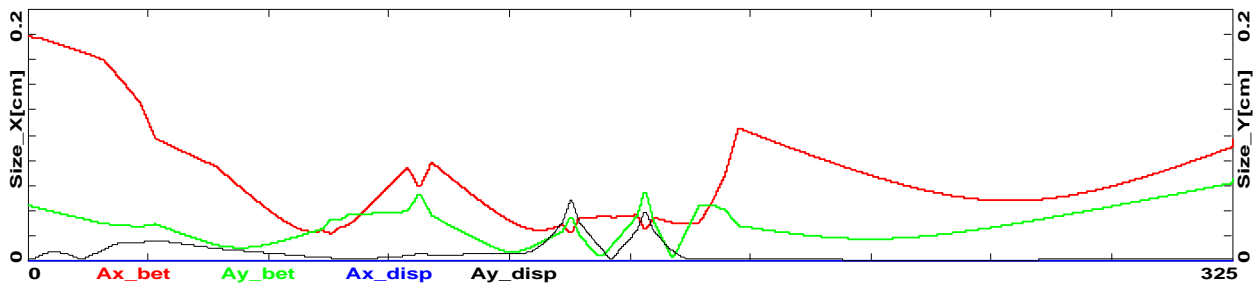
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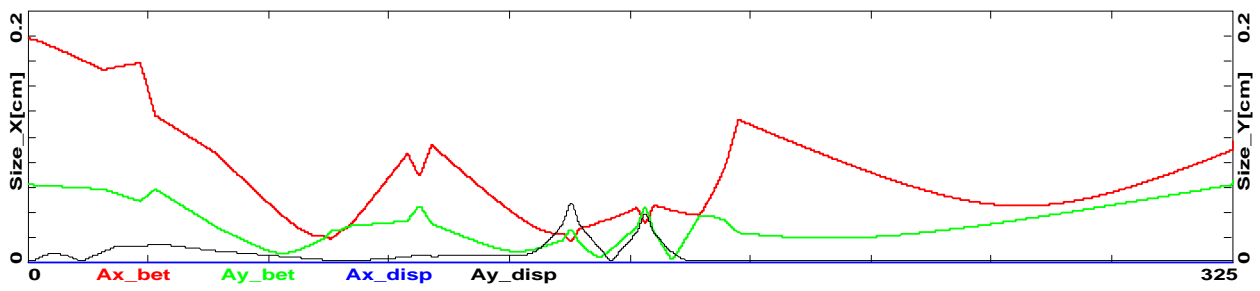
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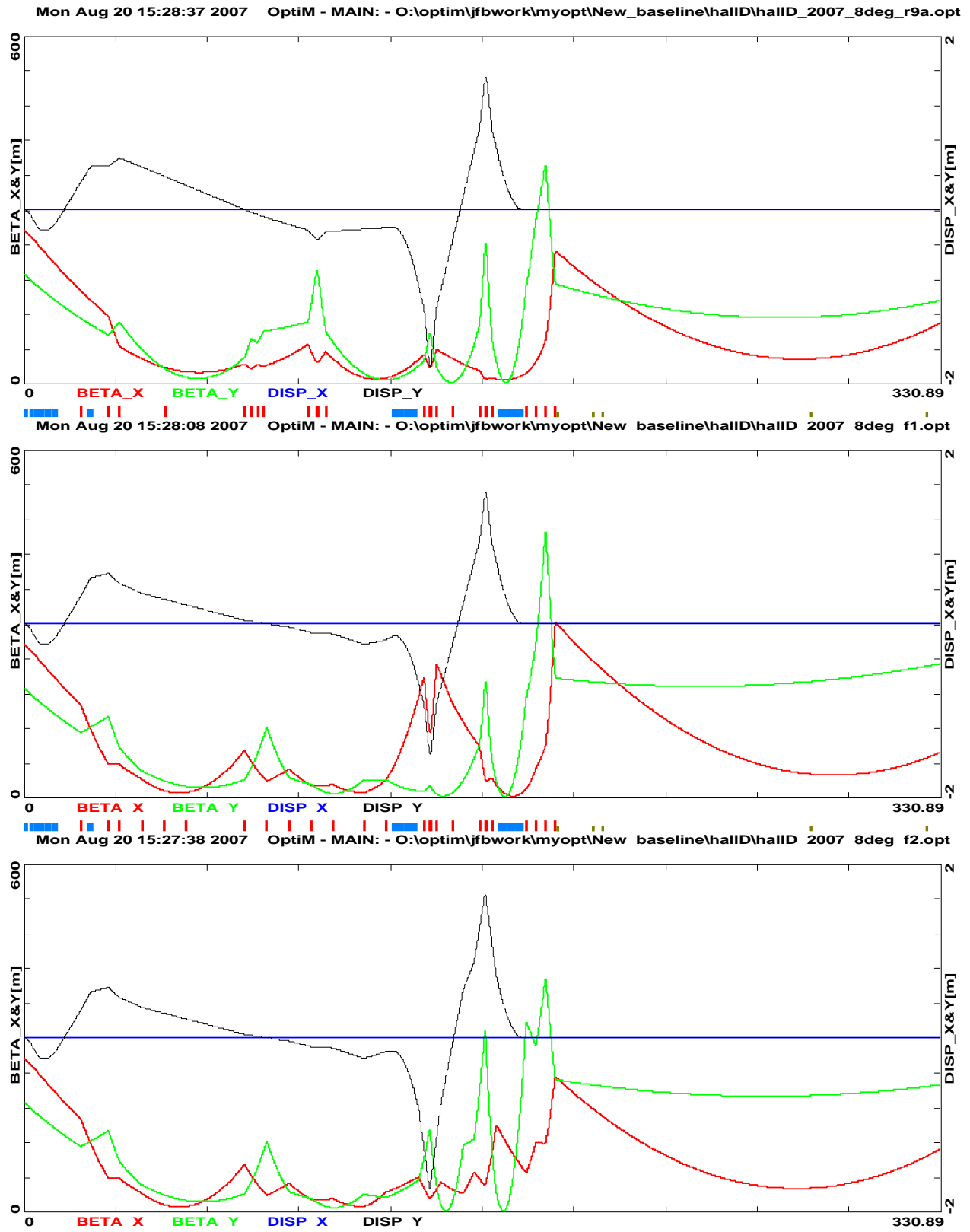
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Five more input parameter variations

quad	baseline	T1	T2	T3	T4	T5	T6	T7	T8	T9	max_amp
MQCBS01	-168	-168	-4113	361	-794	-168	-2397	2196	5852	-5438	5853
MQCBS02	19066	19066	19067	23730	20669	19066	23281	18051	11425	28137	28138
MQCBS03	-24944	-24944	-20126	-33637	-26877	-24944	-28948	-26734	-23484	-29874	33638
MQCBS04	-233	-233	0	23500	5315	-233	8416	3667	6857	5193	23500
MQPBE01	34843	34843	34804	6490	35661	34843	34353	37027	37994	34886	37994
MQPBE02	-49718	-49718	-49753	-29309	-47198	-49718	-49273	-47267	-47312	-49906	49907
MQPBE03	30955	30955	30906	18300	31769	30955	29374	32485	31333	29365	32486
MQPBE04	-18969	-18969	-16016	-7803	-17626	-18969	-20850	-18082	-20697	-21946	21946
MQPBT01	45810	45810	45810	46284	45810	45810	45810	45810	45810	45261	46284
MQPBT02	-39663	-39663	-39663	-39663	-39663	-39663	-39663	-39663	-39663	-39395	39664
MQPBT03	-39663	-39663	-39663	-39663	-39663	-39663	-39663	-39663	-39663	-39395	39664
MQPBT04	46079	46079	46079	46284	46079	46079	46079	46079	46079	45526	46284
MQW5C01	72315	72315	74889	72294	72315	72315	72315	72315	72315	72358	74889
MQW5C03	-76623	-76623	-77648	-76614	-76623	-76623	-76623	-76623	-76623	-76639	77649
MQW5C04	-76623	-76623	-77648	-76614	-76623	-76623	-76623	-76623	-76623	-76639	77649
MQW5C06	72315	72315	74889	72294	72315	72315	72315	72315	72315	72358	74889
MQP5C08	2142	2142	36494	11694	8993	2142	8980	2128	8946	7589	36495
MQW5C15	72020	72020	82366	71901	71881	72020	72070	72023	71882	71275	82367
MQW5C16	-80731	-80731	-84514	-80733	-80731	-80731	-80804	-80731	-80731	-80493	84515
MQW5C17	-80731	-80731	-84514	-80733	-80731	-80731	-80804	-80731	-80731	-80493	84515
MQW5C19	72020	72020	82366	71901	71881	72020	72070	72023	71882	71275	82367
MQW5C20	-19942	-35621	35281	-55209	-19846	-12643	-27533	-50453	-55453	-55859	55859
MQW5C23	-8370	-10535	-82688	17436	-8234	-5716	-6294	-15905	-12199	-10409	82689
MQW5C24	-54469	-42342	34692	61225	-54349	-59118	-51134	-26976	-23256	-21861	61225
MQW5C26	54414	52061	6411	-47302	54051	56202	51878	48312	44610	42437	56203

Quad settings for ten cases of rev 9a and max amplitude for each. QC has max of 40000, QP of 75000 and QW of 100000, all with 20A power supplies. The quads in this table were originally numbered in sequence. During the development of the 8 degree design many quads were found to be unnecessary and were removed. The remaining quads were renumbered. Pairs of quads which serve as the middle elements of triplets, for instance BT02/BT03 and 5C03/5C04, are not numbered in the usual ##A sequence because when I started it wasn't clear whether they would be part of triplets or quadruplets.



Top plot is nominal optics, second has relatively even quad spacing before the ramp and third is as even as I could get it throughout. These plots are beta functions and dispersion, not envelopes.

IV. Searching for an alternate scheme (Yves Roblin)

Systematic studies were carried out to search for an alternate scheme for the hall D beam line. The idea behind the study was to try to use standard building blocks in order to decouple the various functions that one has to have. Ideally, a well-defined set of quadrupoles would be used for the betatron matching into the ramp, another set for setting the spot size and beam divergence at the target. Between the spreader and the ramp, one would use a FODO structure to transport the beam. The ramp would be an achromatic beam translator.

The first set of quadrupoles in the spreader are used to match the beam into a FODO structure which is depicted below. This FODO channel is provided by seven equally spaced quadrupoles and carry the beam to the entrance of the ramp.

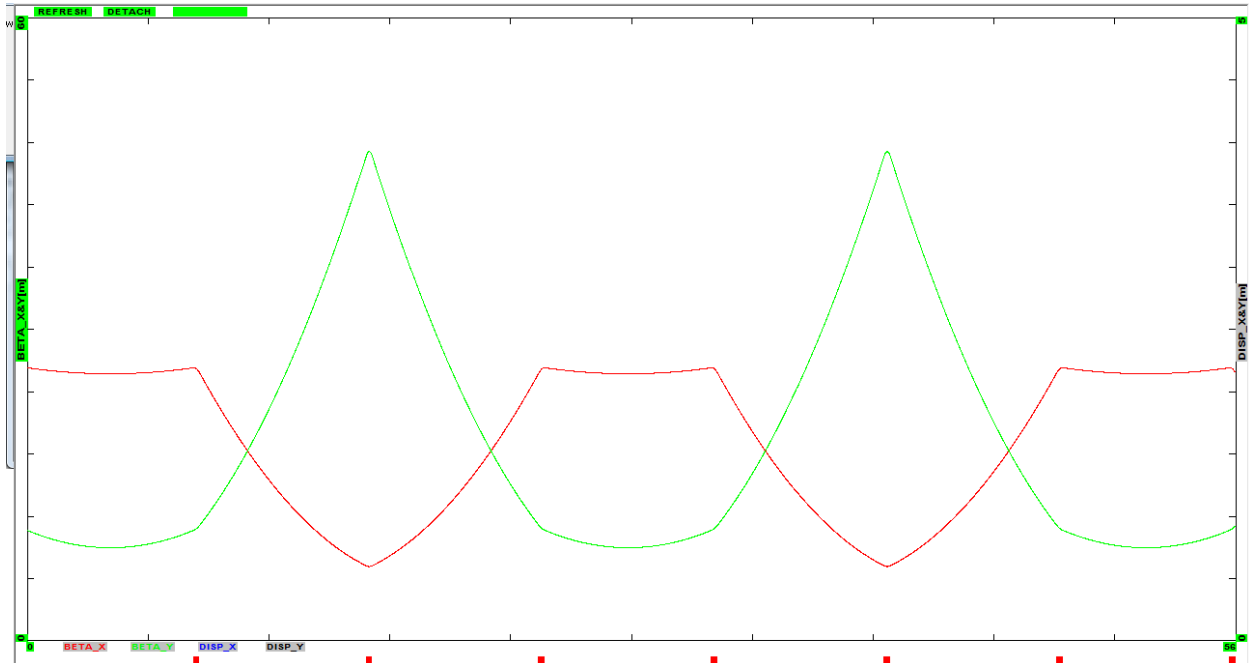


Figure 1: Betatron Functions for FODO structure

These seven quadrupoles are of type MQP. The gradients necessary to produce this FODO structure are well within the capabilities of these quadrupoles.

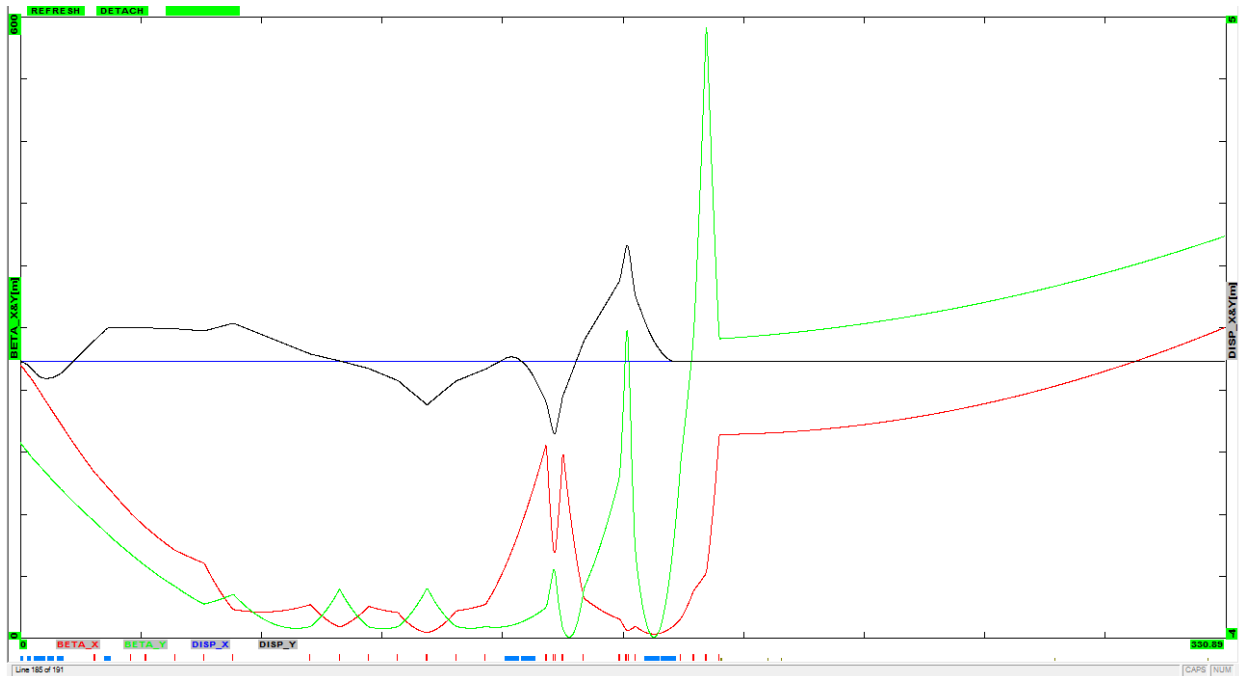


Figure 2: Alternate hall D design

This alternate hall D design is shown in figure 2. The resulting beam sizes are depicted in figure 3.

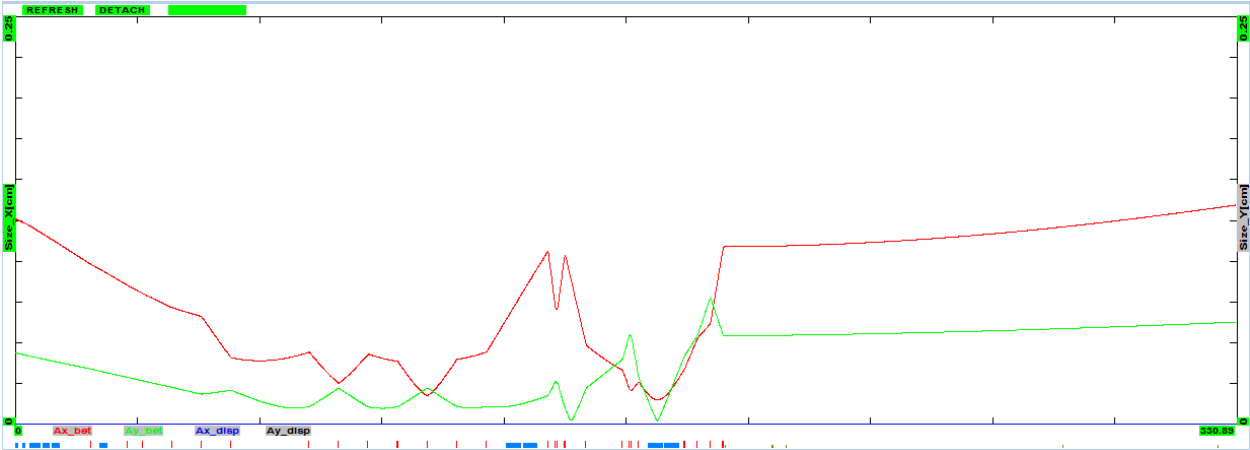


Figure 3: beam sizes in hall D with alternate design

Several features make this design interesting. First, the beam divergence between the radiator and the collimator is very small. Beam sizes are reasonably small (120 microns for x, 60 microns for y) and should yield a relatively halo free beam. The functions are clearly separated. Quadrupoles MQCBS01 thru MQCBS06 are used to match into the FODO structure. The dispersion suppression and betatron control in the ramp is achieved by moving three groups of quadrupoles independently. MQW5C01 together with MQW5C06, MQP5C08 by itself to roll the dispersion and MQW5C03 together with MQW5C04.

Finally, the control of the spot size and divergence at radiator and collimator is done by using a symmetric quadruplet: MQW5C15 moving together with MQW5C19 and MQW5C16 moving together with MQW5C17.

The phase advance is sufficient to allow for fast feedback systems as shown in figure 4.

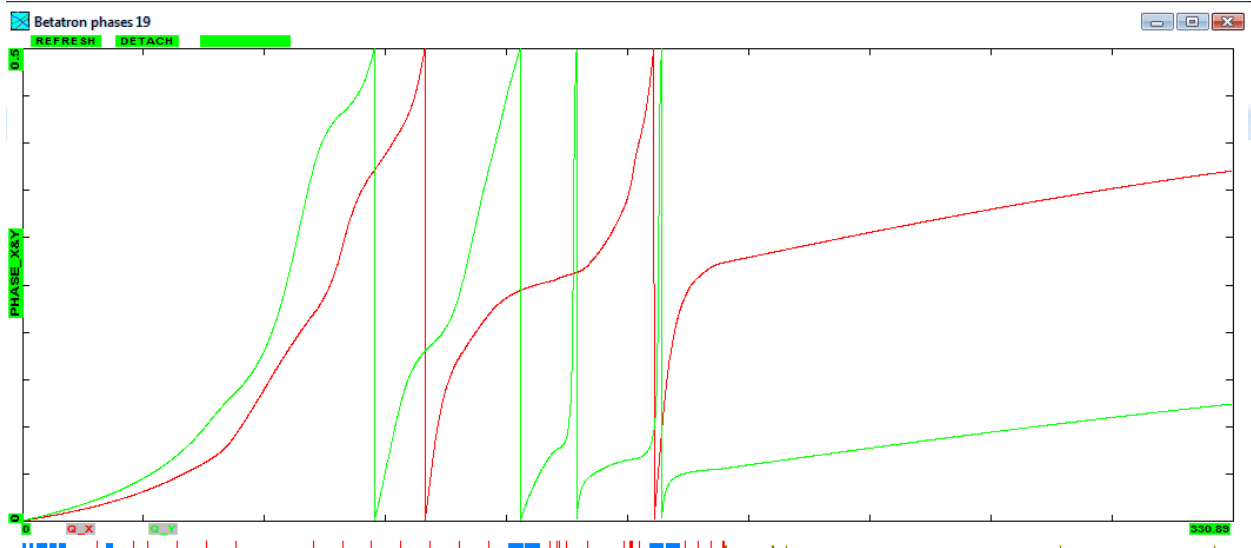


Figure 4: Phase advances in the hall D alternate design

V. The benefits of hot cryomodules at the front of the NL and the DBA optics.

January 11, 2008, Jay was asked to use input Twiss parameters and emittances from one of the improved lattices, that with DBA optics in arcs 6-10 and the hot cryomodules in the front of the NL, as input to the ten cases shown on pages 3 and 4. The next two pages show beam sizes for these cases with the smaller betas and emittances. In most cases the first eight quads were used to match transverse parameters at the start of the ramp to the baseline case. Subsequently three optical elements in the ramp and four after the ramp were used to null dispersion and set the transverse beam size at the collimator as close to nominal as possible. Beam sizes at radiator and collimator are smaller than nominal in all cases. The radiator will have to be moved more often due to damage from the higher power density electron beam. The collimator will admit more of the polarized photons if its size is kept constant. If the collimator aperture is reduced, polarization may be increased slightly as unpolarized photon content is reduced. The required orbit lock precision depends upon collimator size.

Following the ten plots is a table like that on page 5. Gradients in the first quadruplet increase such that QP must replace QC quads; other quads remain close to previous values. Others must decide if the smaller beams allow one to switch to QA quads from QP/QW quads. Jay would not change to the smaller apertures. Recall that the ten cases reflect only the best quartile of matching ever achieved in the machine. The assumption of best quartile would "use up" all the benefit gained from the decreased beam size. An improved process for matching, including necessary additions to diagnostics, is needed.

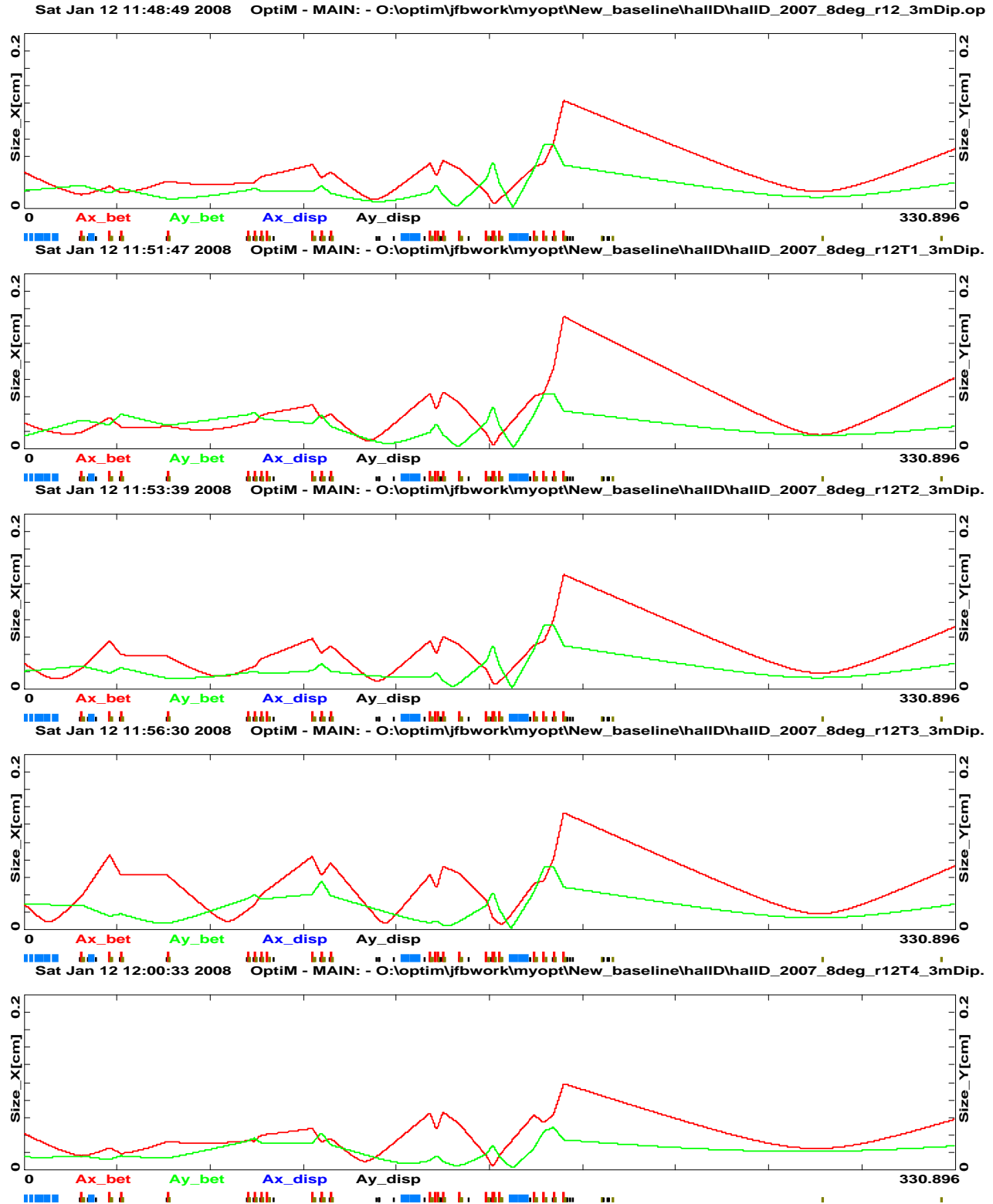
The plots shown assume 3m dipoles at 9.15 kG, BTW. It is hypothesized that the cost of buying four of these, even with tooling costs, is less than the cost of buying four 4m dipoles. Space is available for the 4m dipoles. The exact length of these magnets may be decided later.

Input parameters used in the ten plots which follow are:

BetaX[cm]=6572 BetaY[cm]=6074
 AlphaX=2.36 AlphaY=-0.78
 Emittances: X=2.5 nm, Y = 0.64 nm
 DP/P=2.5e-04

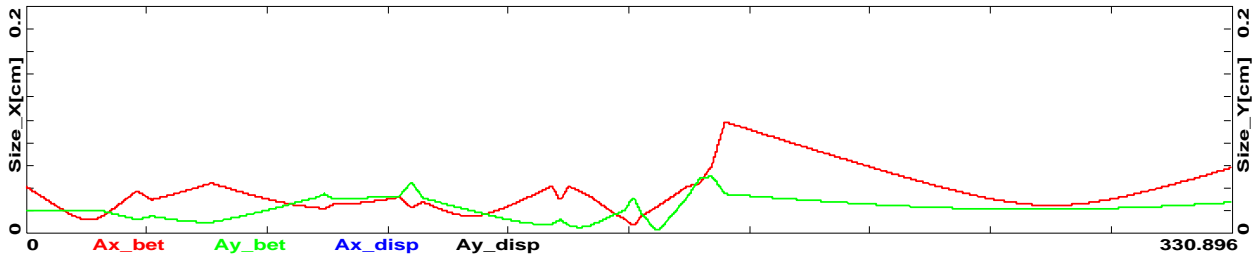
Betas are smaller than the baseline values shown immediately below because putting the hot cryomodules in the front of the NL increases focusing of the higher energy beam in the sixth pass. Emittances are predominantly reduced by the DBA optics.

Baseline values used on pages 1-9: BetaX[cm]=26365 BetaY[cm]=18822.1
 AlphaX=2.16842 AlphaY=-1.72148 Emittances: X=6 nm, Y = 1 nm

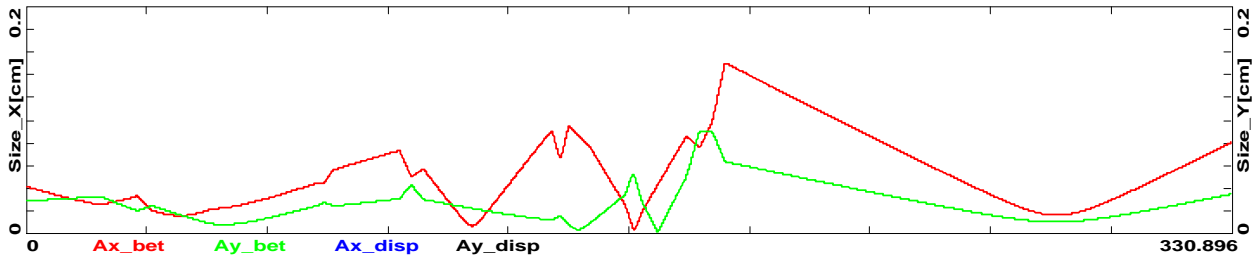


Beam envelopes for nominal and first four variations with improved input parameters. Compare with page 3 which has baseline input parameters. Vertical scale is the same, 2mm.

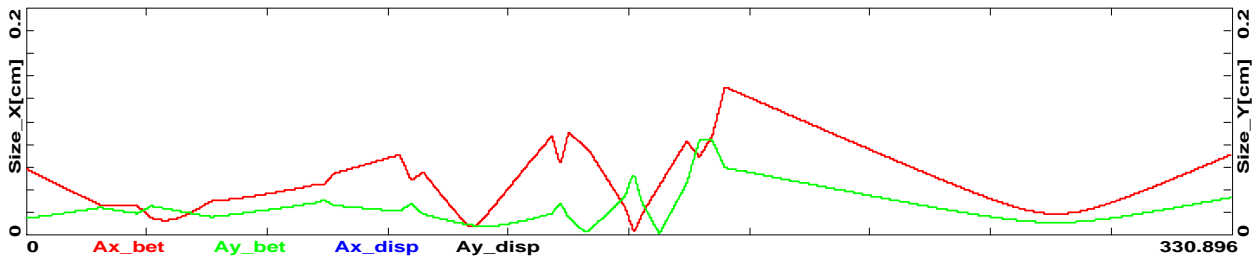
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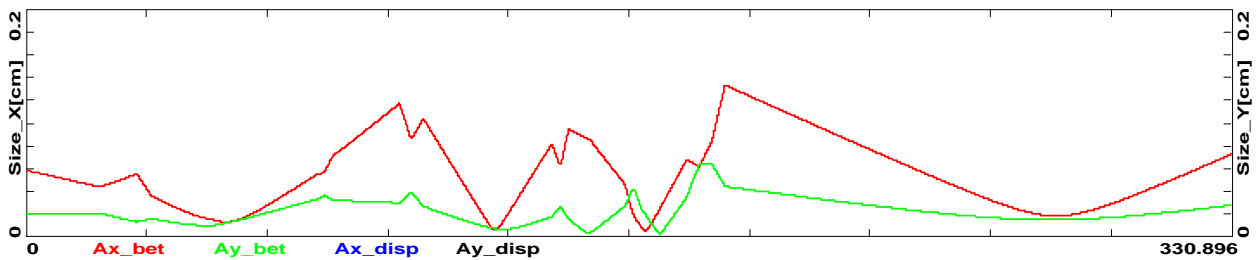
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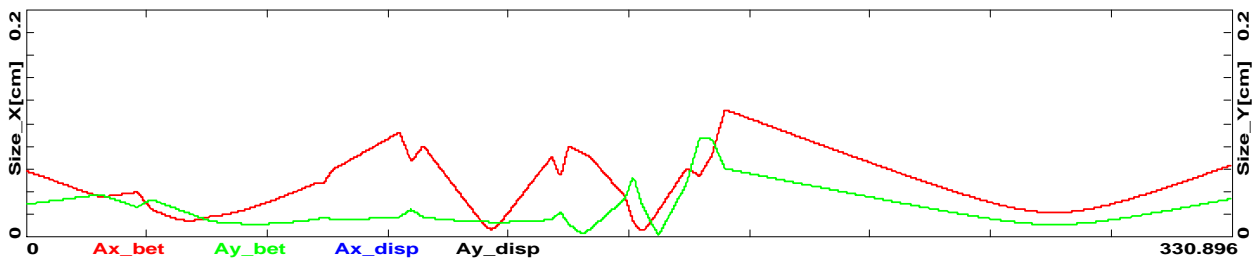
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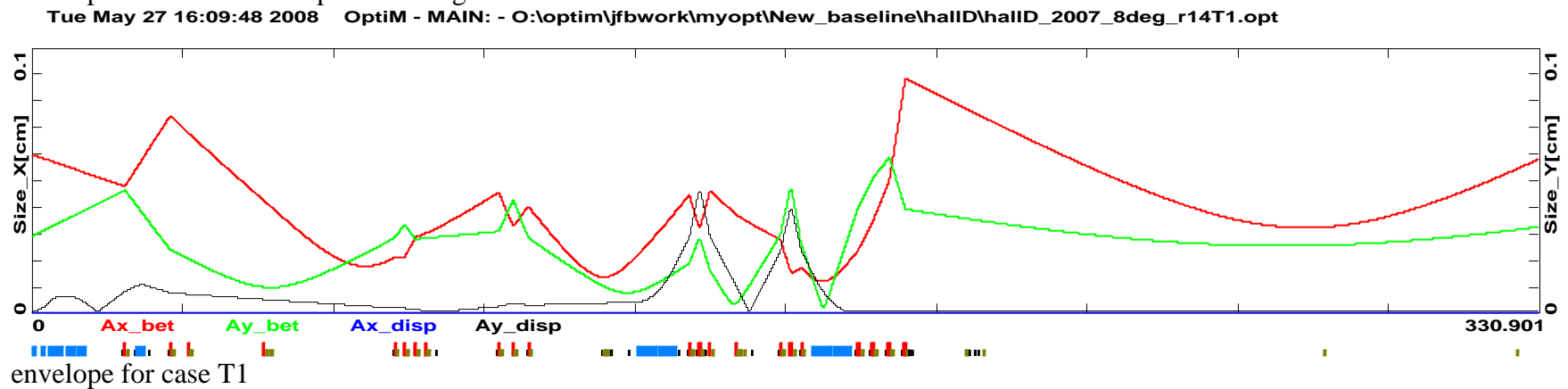
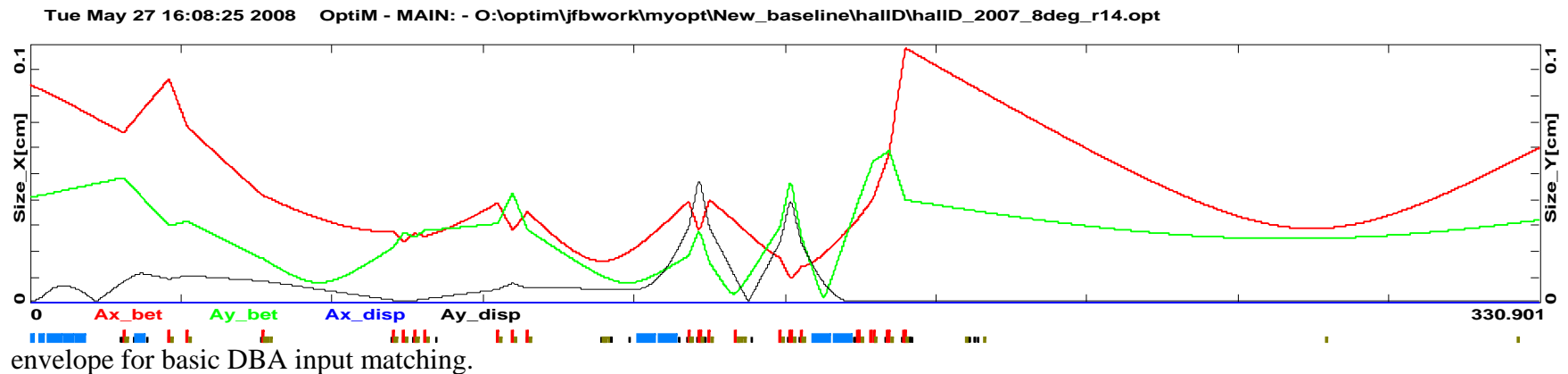
Cases T5-T9 with new input parameters. Compare with page 4.

	nominal	T1	T2	T3	T4	T5	T6	T7	T8	T9	maximum amplitude
MQCBS01.BDL	-19492	-18563	-18484	-19053	-19053	-19053	-19053	-18409	-17394	-17394	19492
MQCBS02.BDL	53223	54345	53955	50802	50802	50802	50802	48276	44826	44826	54345
MQCBS03.BDL	-39583	-40365	-40167	-38036	-38036	-38036	-38036	-37936	-36750	-36750	40365
MQCBS04.BDL	21855	18352	18461	18664	18664	18664	18664	23016	23781	23781	23781
MQPBE01.BDL	6723	6405	6481	7209	7209	7209	7209	7280	8232	8232	8232
MQPBE02.BDL	-39924	-39051	-39087	-39246	-39246	-39246	-39246	-39580	-39298	-39298	39924
MQPBE03.BDL	24724	26645	26525	25755	25755	25755	25755	24870	23949	23949	26645
MQPBE04.BDL	577	3730	3492	1795	1795	1795	1795	443	-1498	-1498	3730
MQPBT01.BDL	45810	45982	45982	45982	45982	45982	45982	45982	43716	43716	45982
MQPBT02.BDL	-39556	-38169	-38169	-38169	-38169	-38169	-38169	-38169	-37354	-37354	39556
MQPBT02A.BDL	-39556	-38169	-38169	-38169	-38169	-38169	-38169	-38169	-37354	-37354	39556
MQPBT03.BDL	46079	46251	46251	46251	46251	46251	46251	46251	43972	43972	46251
MQW5C01.BDL	72315	72315	72315	72315	72315	72315	72315	72315	72315	72315	72315
MQW5C02.BDL	-76583	-76583	-76583	-76583	-76583	-76583	-76583	-76583	-76583	-76583	76583
MQW5C02A.BDL	-76583	-76583	-76583	-76583	-76583	-76583	-76583	-76583	-76583	-76583	76583
MQW5C03.BDL	72315	72315	72315	72315	72315	72315	72315	72315	72315	72315	72315
MQP5C04.BDL	12067	11982	12029	12041	12041	12041	12041	12046	12036	12036	12067
MQW5C05.BDL	73799	73832	73813	73813	73813	73813	73813	73905	74110	74110	74110
MQW5C06.BDL	-81497	-81502	-81502	-81502	-81502	-81502	-81502	-81537	-81612	-81612	81612
MQW5C06A.BDL	-81497	-81502	-81502	-81502	-81502	-81502	-81502	-81537	-81612	-81612	81612
MQW5C07.BDL	73801	73829	73825	73821	73821	73821	73821	73913	74107	74107	74107
MQW5C08.BDL	17133	17150	17749	26655	40463	14527	40412	45445	46314	46313	46314
MQW5C09.BDL	-44404	-44476	-43741	-46411	-39356	-36473	-51506	-53936	-55644	-54905	55644
MQW5C10.BDL	-37059	-37175	-37319	-37252	-46107	-41800	-35010	-34242	-34751	-34327	46107
MQW5C11.BDL	49878	49672	50019	49971	45880	49807	45149	43650	47153	46104	50019

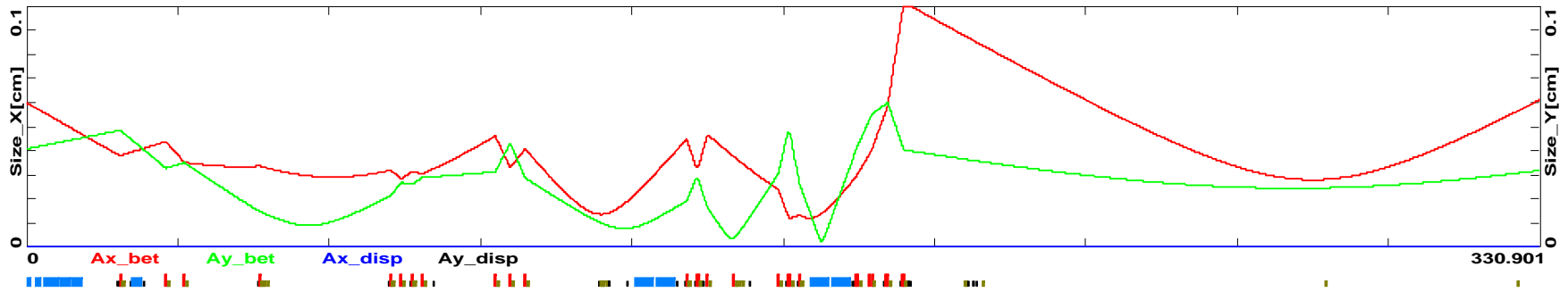
Quad values for ten cases shown on previous two pages. Two quads are highlighted because the values exceed those possible for QCs. The four QC quads at the front of the line must be changed to QP quads, which have adequate capacity, due to the smaller incoming beam. If others judge the beam small enough throughout, the QPs and QWs which follow these four, except for the last quadruplet, **might** be replaced with QA quads. The paired BT 02/2A could be replaced with a single QA. The paired quads in the ramp, 2/2A and 6/6A, cannot be replaced with a single QR as the latter doesn't provide enough gradient.

VI. DBA optics as input to hall line

In this section I used Twiss parameters obtained from Yves Roblin for the DBA optics as input to the B spreader. Alphas and betas are close to baseline (page 10); emittances are 3nm and 0.9nm (x,y). (*The sequence in this note is chronological.*) I show the same ten variations of input conditions. I have been inconsistent in these plots in the inclusion of the vertical beam size due to dispersion. In the first set of graphs I show the dispersive component (black) consistently. In the second set I show only the betatron envelope contributions, horizontal (red) and vertical (green). In this set I show the dispersive piece in some graphs and not in others - I was optimizing in two Optim sessions simultaneously and turned it off in one of the output windows. Maximum vertical size in the ramp is under 0.75mm. Maximum horizontal size at the radiator is 1.2mm; 1.5 mm is allowable. QA beam pipe radius is 11mm. QP beam pipe radius is 16mm. Alphas and betas are close to baseline; emittances are 3nm and 0.9nm (x,y). *Vertical scale 1mm, not 2mm.*

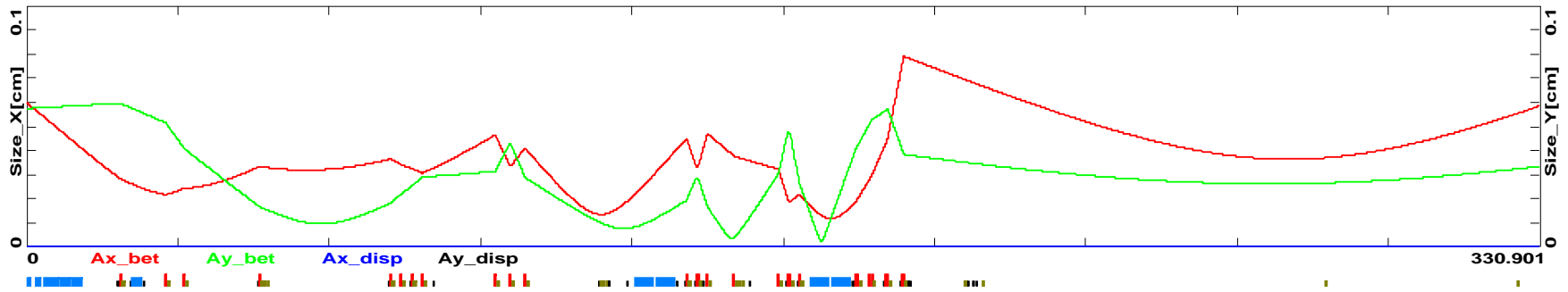


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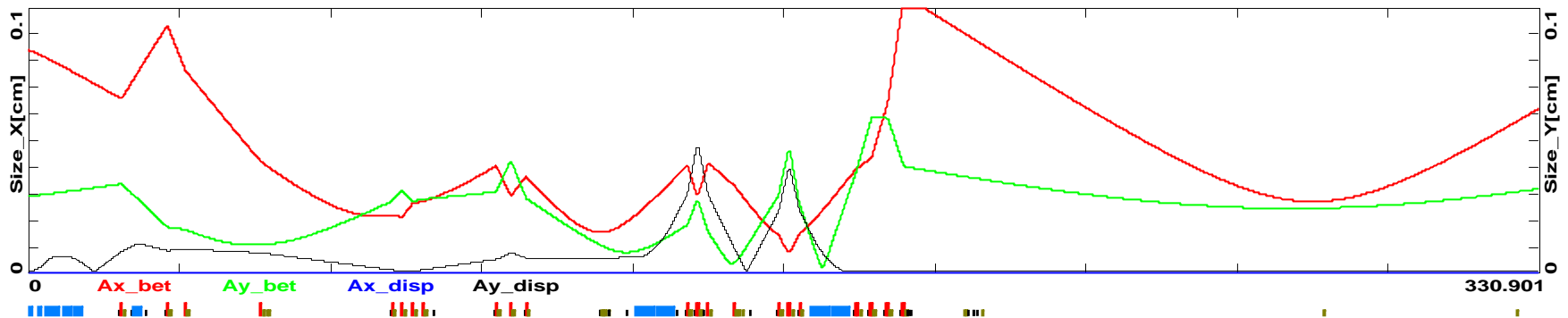
envelope for case T2

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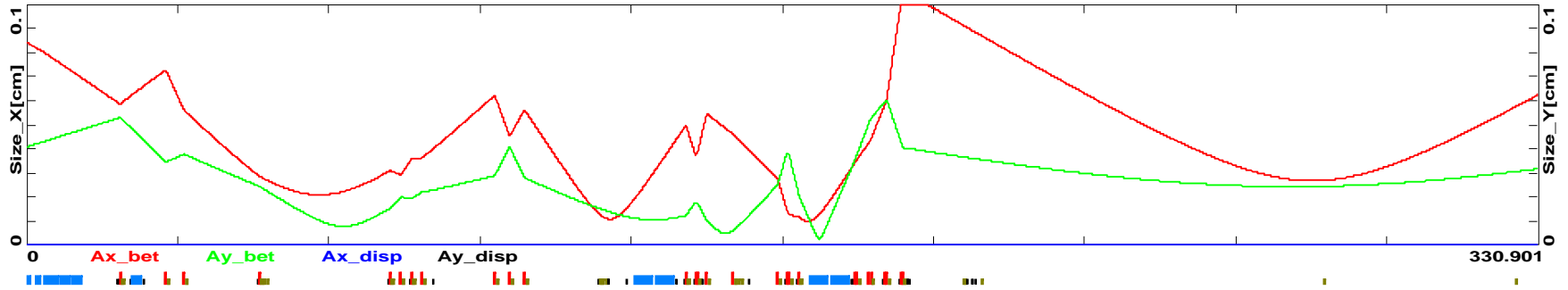
envelope for case T3

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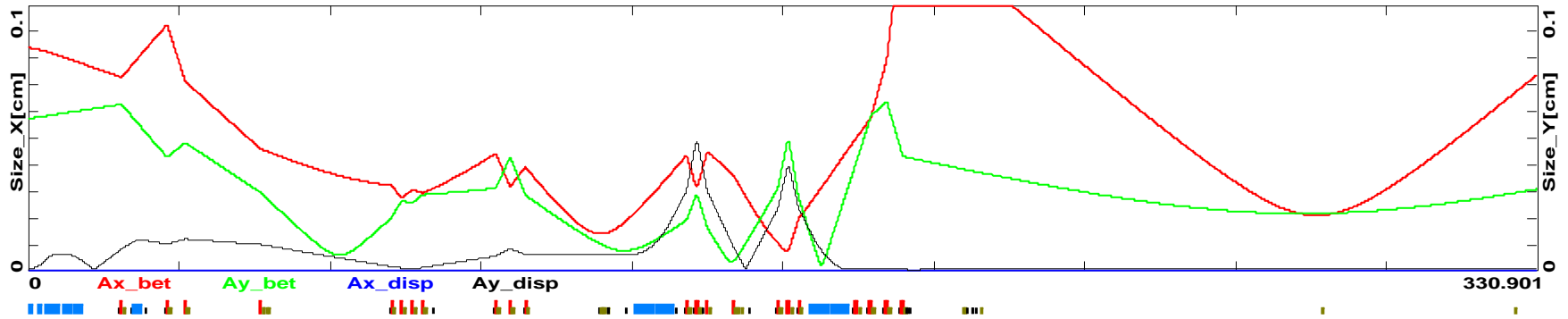


envelope for case T4. Horizontal peak in last quadruplet less than 1.1 mm.

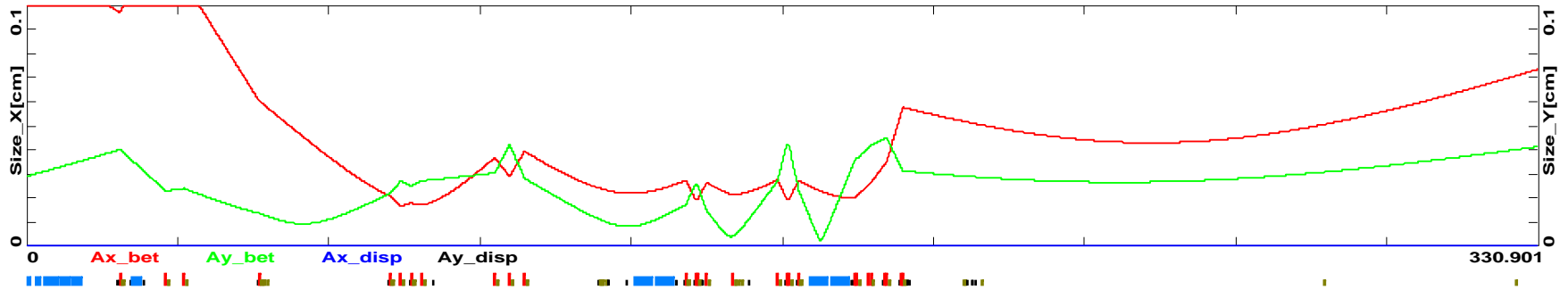
Tue May 27 16:40:35 2008 OptiM - MAIN: - O:\optim\jfbwork\myopt\New_baseline\halID\halID_2007_8deg_r14T5.opt



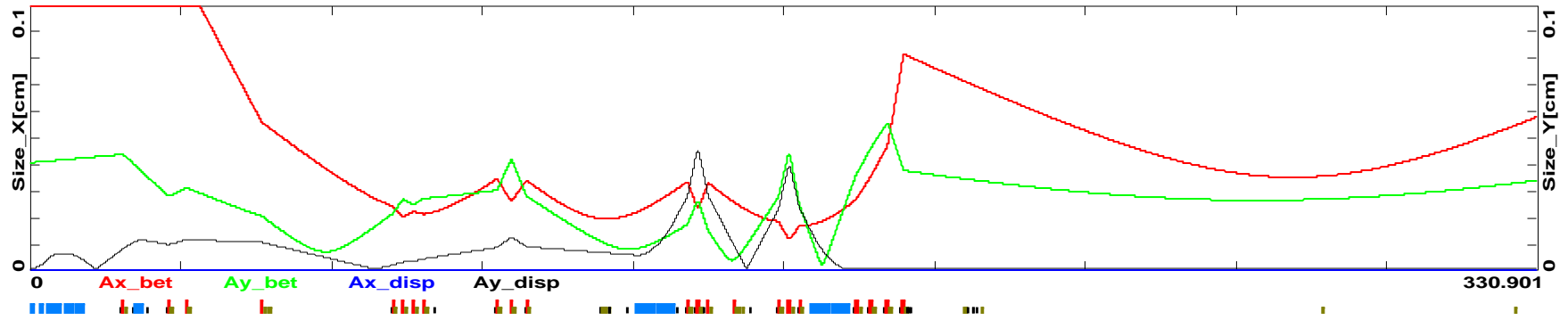
Tue May 27 17:00:03 2008 OptiM - MAIN: - O:\optim\jfbwork\myopt\New_baseline\halID\halID_2007_8deg_r14T6.opt



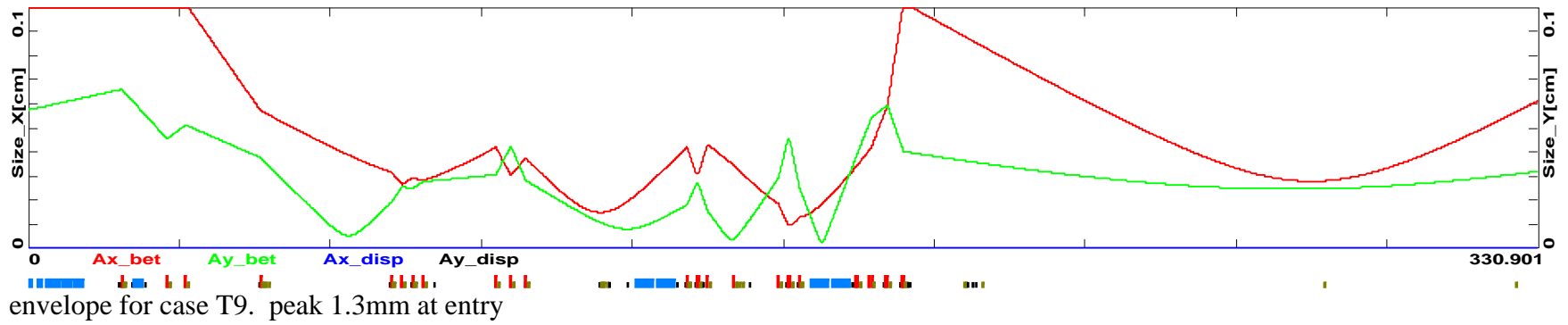
Tue May 27 17:16:14 2008 OptiM - MAIN: - O:\optim\jfbwork\myopt\New_baseline\halID\halID_2007_8deg_r14T7.opt



Tue May 27 17:38:38 2008 OptiM - MAIN: - O:\optim\jfbwork\myopt\New_baseline\halID\halID_2007_8deg_r14T8.opt



Tue May 27 17:42:59 2008 OptiM - MAIN: - O:\optim\jfbwork\myopt\New_baseline\halID\halID_2007_8deg_r14T9.opt



Quad values with DBA input. Here the BT02/BT02A quads in the previous table have been combined into one QA. It appears that the 75000 G QP limit suffices for the last four girders. It still would be good to have girders capable of accepting double quads there. The quads with 0 for all cases represent the empty slots. 8A is empty rather than 8 because the 5C08 girder would hit the ramp dipole if it were extended upstream rather than downstream. In the other three other cases upstream extension is preferred mechanically.

magnet	rev14	T1	T2	T3	T4	T5	T6	T7	T8	T9	max ampl
MQPBS01.BDL	-19026	-27659	-16575	-6328	-23283	-19026	-14753	-23438	-16383	-15389	27659
MQPBS02.BDL	31646	23806	26499	-14828	30660	31646	32098	34672	34115	31520	34672
MQPBS03.BDL	-17978	-627	-21533	11880	-11494	-17978	-21382	-17025	-19353	-18256	21533
MQPBS04.BDL	-6875	991	7822	14180	-7769	-6875	-7002	-9581	-13781	-11771	14180
MQABE01.BDL	25912	19306	26811	17514	15318	25912	24226	24136	19932	20711	26811
MQABE02.BDL	-49667	-58264	-41786	-4250	-48296	-49667	-49342	-48601	-48776	-51505	58264
MQABE03.BDL	30209	39793	24497	2844	33921	30209	27445	31180	29305	26759	39793
MQABE04.BDL	-14499	-1679	-15996	-21465	-3866	-14499	-17778	-12090	-12297	-16886	21465
MQABT01.BDL	45810	45810	45810	45810	45810	45810	45810	45810	45810	45810	45810
MQABT02.BDL	-77815	-77815	-77815	-77815	-77815	-77815	-77815	-77815	-77815	-77815	77815
MQABT03.BDL	46079	46079	46079	46079	46079	46079	46079	46079	46079	46079	46079
MQA5C01.BDL	72705	72705	72705	72705	72705	72705	72705	72705	72705	72705	72705
MQA5C02.BDL	-75699	-75699	-75699	-75699	-75699	-75699	-75699	-75699	-75699	-75699	75699
MQA5C02A.BDL	-75699	-75699	-75699	-75699	-75699	-75699	-75699	-75699	-75699	-75699	75699
MQA5C03.BDL	72705	72705	72705	72705	72705	72705	72705	72705	72705	72705	72705
MQA5C04.BDL	4289	-5220	-205	-9929	9984	4289	16811	-17480	185	4419	17480
MQA5C05.BDL	72715	74014	73323	74666	73555	72715	74536	75881	73373	72759	75881
MQA5C06.BDL	-79831	-80547	-80248	-80933	-80082	-79831	-80359	-81155	-79961	-79774	81155
MQA5C06A.BDL	-79831	-80547	-80248	-80933	-80082	-79831	-80359	-81155	-79961	-79774	81155
MQA5C07.BDL	72715	74014	73323	74666	73555	72715	74536	75881	73373	72759	75881
MQP5C08.BDL	-4093	-18788	-10883	-19404	9917	4989	1358	-33760	-19489	-1896	33760
MQP5C08A.BDL	0	0	0	0	0	0	0	0	0	0	0
MQP5C09.BDL	0	0	0	0	0	0	0	0	0	0	0
MQP5C09A.BDL	-26795	-9923	-20922	-16491	-43112	-20086	-27682	-8231	-4519	-24939	43112
MQP5C10.BDL	0	0	0	0	0	0	0	0	0	0	0
MQP5C10A.BDL	-45666	-53936	-48253	-47954	-34793	-54558	-47352	-44712	-57136	-48528	57136
MQP5C11.BDL	0	0	0	0	0	0	0	0	0	0	0
MQP5C11A.BDL	52198	54744	53591	54329	49031	54352	51801	49263	53208	52865	54744

VII. Miscellaneous comments not relating to optics

The Optim decks corresponding the solutions in sections V. and VI have been mostly (V) and fully (VI) populated with diagnostics and correctors in line with TN-05-053. Section V shows that 3m dipoles (9.15 kG) would suffice. 2.5m dipoles (11 kG) would be possible but cause more vertical emittance growth. 2m would not be good because the high field (13.72kG) would cause unacceptable levels of multipoles (TN in preparation). Fast feedback correctors were added at the request of the I&C CAM, made 5/21/08; Jay had forgotten their inclusion in the baseline.

VIII. Conclusions

The first author (JFB) is uncomfortable with the assumption that we will always achieve matching in the best quartile of historical CEBAF results because there are no plans to add the arc by arc diagnostics necessary to achieve this result. If one assumes this will occur before the 12 GeV shutdown, the solution shown in section VI, using QP and QA quads, will likely suffice.

Acknowledgment

This note was improved by discussions with Mike Tiefenback.