Simulation Results of Two-Dimensional Multi-Pass Beam Breakup

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Abstract

This paper is intended to serve as a compliment to those results described in Reference [1]. In this paper we present simulation results of the multipass, multibunch beam breakup (BBU) instability in two dimensions for higher-order modes (HOMs) oriented at arbitrary angles and with a full $4\times4$ recirculation matrix. Simulations were performed using a newly developed BBU code. The threshold current formula from the analytical model for a single cavity shows that with a judicious choice of recirculation matrix and HOM polarization, beam breakup can be greatly alleviated. Results of the simulations are shown to be in excellent agreement with the theoretical results. In addition, preliminary results of observations of beam breakup from the JLab 10 kW FEL are discussed which support the theoretical predictions on the effect of HOM orientation on the threshold current.

Introduction

The motivation for considering the problem of beam breakup (BBU) in two dimensions came as a result of observing BBU in Jefferson Lab’s 10 kW FEL due to the new 7-cell cryomodule. Besides the obvious difference in the number of cells, the new 7-cell cavities differed from their 5-cell predecessors in the choice of higher-order mode (HOM) couplers. Whereas waveguide HOM couplers were used in the 5-cell cavities, the new 7-cell cavities use DESY inspired coaxial HOM couplers. Schematics of each type of coupler and their respective HOM port geometries are displayed in Figure 1. Because of the geometry of the waveguide couplers, the HOMs tended to be aligned along the horizontal ($x$) and vertical ($y$) axes and so any analysis of BBU was done by considering each transverse plane independently using the one dimensional BBU formalism. The HOM ports from the coaxial couplers, however, have a different geometry altogether and it was thought that perhaps the HOM polarization would change accordingly. However, it seems likely that the polarization in the 7-cell cavities is determined, in large part, due to cavity imperfections rather than HOM coupler geometry [2]. Nevertheless, the question remains how the threshold current is modified (if at all) by having an HOM oriented at some angle other than along the planes of (decoupled) transverse motion.
Figure 1: Schematic of coaxial-type HOM coupler used on the new 7-cell cavities (left) and the waveguide-type HOM coupler used on the 5-cell cavities (right).

Theory

As described in Reference [1], a novel approach was used in deriving the threshold current by considering the energy deposited by the beam into cavity HOMs. As is the usual practice, we consider only a single cavity and a single beam recirculation. However, instead of considering an HOM oriented at either 0 or 90 degrees, we allow for the HOM to be oriented at an arbitrary angle, $\alpha$, with respect to the $x$-axis. We also allow for a full 4x4 recirculation matrix (i.e. the off-diagonal 2x2 matrices need not be zero). The threshold current is then given by the following expression:

$$ I_{th} = -\frac{2V_b}{Q(R/Q)k\sin(\omega T_c)[M_{12} \cos^2 \alpha + (M_{14} + M_{32}) \sin \alpha \cos \alpha + M_{34} \sin^2 \alpha]} . \quad (1) $$

In addition to being a nice expression for two-dimensional beam breakup, this result contains very real implications concerning the suppression of breakup which will be discussed shortly.

Simulation Code

The simulations were performed using a newly developed BBU code (yet to be named) which handles full two-dimensional particle tracking and has the capability of handling not only decoupled transverse motion, but coupled motion as well. This has given us a powerful tool to study numerous interesting BBU scenarios, namely, the effects of rotated HOMs and of rotated optics on the threshold current. A more thorough and detailed description of the code is given in Appendix A.
Simulation Results

Before turning to the two-dimensional results, we consider the time and frequency domain behavior of beam breakup in a one-dimensional case. The results are shown in Figure 2. The top plot shows the beam displacement as a function of time (axes have been suppressed for clarity). This is the usual output for the BBU simulation program, and when the displacement is neither growing nor damping, is used to define the threshold current. In this example, the total simulation time was 5 ms and the threshold current has clearly been exceeded as the beam motion is beginning to grow. The system simulated consisted of a single cavity with two HOMs and a single recirculation. To see how the picture of beam breakup evolves in the frequency-domain, Fast-Fourier Transformations (FFTs) were performed on “slices” of the time-domain data (see Figure 2). The FFT plots reveal that initially the frequency of both HOMs are present in the beam but as time progresses one frequency gradually decreases until the breakup develops, at which point only a single driving frequency appears – the “killer-mode”.

![Figure 2: The evolution of BBU in the time and frequency domains. The top plot shows the beam displacement as a function of time for $I_b > I_{th}$. The lower plots show FFTs of the indicated “slices” of the beam displacement. The left and right peaks correspond to frequencies 2110 MHz and 2100 MHz, respectively.](image-url)
I. Single Cavity and Recirculation, One HOM, Decoupled Transverse Motion

We now consider two-dimensional BBU, namely, a single cavity containing a single rotated HOM and a single recirculation. Specifically, we investigate the case in which motion in the horizontal plane is always stable independent of the current (i.e. $I_{th} = \infty$) while the vertical motion exhibits instability when the current exceeds the threshold current ($I_h > I_{th}$). We consider only decoupled transverse motion so the off-diagonal 2x2 matrices of the 4x4 recirculation transfer map are zero. For these particular simulations, we set $M_{34}$ to 3 m and vary the $M_{12}$ element as indicated in Figure 3. The simulations show excellent agreement with the theoretical predictions. The asymptotic behavior can be understood by considering equation (1) for decoupled motion (i.e. $M_{14} = M_{34} = 0$) which shows that the threshold current is infinity when the following condition is met

$$\tan^2 \alpha = -\frac{M_{12}}{M_{34}}$$

(2)

Figure 3: Effect of HOM orientation on the threshold current for a single cavity, with a single HOM and a decoupled recirculation matrix. The different traces show the effect of changing the $M_{12}$ element of the recirculation matrix.

To see the effect of using a full 4x4 recirculation matrix and to verify the theoretical results as expressed in equation (1), simulations were performed for two specific cases. For each case, the threshold current was determined as a function of the HOM orientation with respect to the x-axis. Each case used a single cavity containing one HOM - the only difference was in the recirculation matrix.
II. Single Cavity and Recirculation, One HOM, Decoupled Transverse Motion

In this case, the decoupled recirculation matrix given below was used:

\[
\begin{pmatrix}
1.100 & 2.000 & 0 & 0 \\
0.325 & 1.500 & 0 & 0 \\
0 & 0 & 0.900 & 3.000 \\
0 & 0 & -0.063 & 0.900
\end{pmatrix}
\]  \hspace{1cm} (3)

The particular form of the matrix (namely, \(M_{12}\) and \(M_{32}\) are greater than zero) ensures that the beam will be unstable above some threshold current regardless of the HOM orientation. The results of the simulation are displayed in Figure 4.

![Threshold current versus HOM orientation for a single cavity with a one HOM and a decoupled recirculation matrix given by equation (3). For \(I_b > I_{th}\) beam is unstable for all possible HOM orientations.](image-url)
III. Single Cavity and Recirculation, One HOM, Coupled Transverse Motion

For this case the coupled recirculation matrix given below by equation (4) was used. This matrix was generated by rotating the matrix given in equation (3) by 90 degrees

\[
\begin{pmatrix}
0 & 0 & 0.900 & 3.000 \\
0 & 0 & -0.063 & 0.900 \\
-1.100 & -2.000 & 0 & 0 \\
-0.325 & -1.500 & 0 & 0
\end{pmatrix}
\]  
(4)

The results of this simulation are shown in Figure 5. In this case there are well defined regions of beam stability. For particular regions of HOM orientation, the beam is stable independent of the current. For both cases, the simulation shows excellent agreement to the theoretical predictions.

Figure 5: Threshold current versus HOM orientation for a single cavity with one HOM with a coupled recirculation matrix given by equation (4). The beam is unstable for particular regions of HOM orientation and unstable in others.
IV. BBU Suppression by Means of Rotating Optics

In recent months there has been a renewed interest in Reference [3] in which the authors outline means of suppressing BBU by rotating the betatron planes by 90 degrees in the recirculator. The effect of this rotation for a 2-pass system is to effectively break the feedback loop formed between the beam and cavity HOM so there can be no exchange of energy. The idea is conceptually simple. If on the first pass there is an offending mode which produces a deflection in the horizontal plane, then on the second pass (and after a 90 degree rotation), the resultant displacement will be in the vertical plane and will couple to a mode other than the one that caused the deflection. However since the dangerous dipole modes come in pairs, the rotation scheme is effective only if one of the modes is dominant.

As an academic exercise, we have generated an appropriate set of optics for using a solenoid in the FEL Upgrade to accomplish a 90 degree rotation of the betatron planes. Meanwhile a practical implementation of a rotating optics scheme has been worked out for the FEL where they have installed a “skew-quadrupole rotator” in the backleg of the recirculator [5]. To get a feel for the effectiveness of each scheme for suppressing BBU, two-dimensional simulations were performed. The model used for the simulation accurately reflects the machine optics, accelerating cavities (including the effects of RF focusing) and uses measured data of the HOMs. The only unknown is the polarization of each HOM - which were set rather arbitrarily. Therefore, the results of these simulations should not be considered as accurately predicting the performance of the rotating optics in the machine itself. The results are displayed in Figure 6. The top plot shows the beam displacement in each transverse plane as a function of time. For this configuration, the threshold current is 7.3 mA while the average beam current used for the simulation was 20 mA. The middle plot shows the same information but reflects the use of the “skew-quadrupole rotator”. The rotation is clearly effective in suppressing BBU. The new threshold current becomes 263.0 mA – an increase by a factor of ~ 35 in the threshold. The bottom plot reflects the use of a solenoid which also effectively suppresses BBU – however, not by nearly as much as the skew quadrupoles. The new threshold current in this instance becomes 24.3 mA. This rather meager increase in the threshold current is indicative of the fact that both planes have dangerous modes so that a rotation does not necessarily break the feedback of the energy exchange between beam and HOM. (e.g. a rotation in simulations for which HOMs were assigned polarizations in only one direction causes the new threshold current to be greater than 10^6 mA with no indications of breakup).

The point of the previous digression is to point out that schemes are being worked out to rotate the beam to suppress BBU, yet in light of equation (1), one could achieve the same effect by rotating the orientation of the HOMs. Consider for a moment the FEL Upgrade. Although it is nontrivial to extract the $M_{12}$ and $M_{34}$ elements of the recirculation
Figure 6: Simulation results for a distribution of HOMs in Zone 3 of the JLab FEL Upgrade. All simulations use an average beam current of 20 mA and correspond to: BBU with nominal recirculation matrix (top), suppression of BBU with skew-quadrupoles (middle) and a solenoid (bottom) in the recirculator.
transfer matrix at each point along the linac, the two elements are roughly comparable in magnitude and of opposite sign. This suggests that if one could ensure that all the cavity HOMs were oriented at an angle of 45 degrees and that the beam motion is transversely decoupled (a good approximation), then the threshold current would tend to infinity (see equation (2)).

Experiment

Despite limited beam studies time, we have been able to make some revealing measurements regarding the nature BBU at JLab’s 10 kW FEL. A thorough discussion of the measurements, analysis and results will be dealt with another time. However, a few preliminary results are in order as they relate directly to the topic of this paper.

Reference [6] gives a detailed description of the measurements performed on the 7-cell cryomodule and how that data was used in MATBBU simulations to predict the threshold currents due to particular HOMs. The results of the simulations are displayed in Table 1 which was taken directly from the aforementioned reference. One of the limitations of the simulation code is that it can handle only HOMs oriented at either 0 or 90 degrees (note: even if the code were able to accept intermediate angles, as of yet, we have no knowledge of how the HOMs are oriented within each cavity). In light of the implications from equation (1), it is clear that threshold current predictions may be significantly different than in reality. We saw evidence of that on May 27, 2004 when we observed the multipass, multibunch beam breakup instability at the FEL for the first time. Leaving the details for a future discussion, suffice it to say that using Schottky diodes on each of the two HOM ports per cavity (8 cavities x 2 ports = 16 diodes total) we were able to monitor the HOM power levels from each cavity. At the onset of the instability we saw the HOM power levels in cavity 4 (of Zone 3) grow exponentially until the beam tripped off the machine. Repeated BBU induced trips continued to be caused by cavity 4, leaving no doubt that the “killer-mode” was located in this cavity. Initially we were puzzled why we did not see BBU induced trips coming from cavity 7 - at a lower average current - as predicted by simulation (see Table 1). However, we now attribute this to the fact that the predicted killer-mode in cavity 7 is not oriented along the y-axis as we had assumed in the simulation. Of course, the difficulty remains to determine the polarization of a particular HOM in a cavity. Given beam studies time, we may be able to perform some beam based measurements to give us a measure of the HOM polarization axis [7] and confirm with quantitative measurements, the effect on the threshold current.

In the meantime, the best we can do is to study this effect through simulations. Figure 5 shows the dependence of the threshold current due to the 2106.007 MHz mode in Cavity 7 of Zone 3 as a function of HOM orientation. When the mode is oriented along the y-axis (90 degrees) the threshold current is the same as given in Table 1. However, the threshold quickly grows as the mode orientation changes. As discussed in the previous section, the $M_{12}$ and $M_{34}$ elements are comparable in magnitude and so we see the expected effect that the threshold current goes to infinity at approximately 45 degrees in accordance with equation (2).
Table 1: Summary of a MATBU simulation showing mode properties of those HOMs which are predicted to produce threshold currents below 10 mA in the JLab FEL Upgrade [6].

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Loaded Q (Ω)</th>
<th>(R/Q) (Ω)</th>
<th>Threshold Current (mA)</th>
<th>Orientation</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>2102.607</td>
<td>2.61 x 10⁶</td>
<td>29.90</td>
<td>7.07</td>
<td>x-axis</td>
<td>Cavity 8</td>
</tr>
<tr>
<td>2104.683</td>
<td>1.94 x 10⁶</td>
<td>29.90</td>
<td>7.86</td>
<td>x-axis</td>
<td>Cavity 5</td>
</tr>
<tr>
<td>2106.007</td>
<td>6.11 x 10⁶</td>
<td>29.90</td>
<td>2.85</td>
<td>y-axis</td>
<td>Cavity 7</td>
</tr>
<tr>
<td>2114.156</td>
<td>5.21 x 10⁶</td>
<td>28.80</td>
<td>3.68</td>
<td>x-axis</td>
<td>Cavity 4</td>
</tr>
<tr>
<td>2115.201</td>
<td>2.17 x 10⁶</td>
<td>28.80</td>
<td>8.28</td>
<td>y-axis</td>
<td>Cavity 6</td>
</tr>
<tr>
<td>2116.055</td>
<td>3.06 x 10⁶</td>
<td>28.80</td>
<td>4.99</td>
<td>x-axis</td>
<td>Cavity 1</td>
</tr>
<tr>
<td>2116.585</td>
<td>6.66 x 10⁶</td>
<td>28.80</td>
<td>4.18</td>
<td>x-axis</td>
<td>Cavity 7</td>
</tr>
</tbody>
</table>

Figure 7: The threshold current for the 10 kW FEL as a function of the polarization of mode 2106.007 MHz located in Cavity 7 of Zone 3. (Note: the line connecting points is intended only to guide the eye and is not representative of the theoretical prediction as in Figure 3).
Conclusions

The main result from this paper is the excellent agreement shown between simulation results and theory as given by the modified equation for the threshold current expressed in equation (1). Preliminary experimental data also corroborates the theory. The implications of this new theory (and the simulations) are substantial. Given an appropriate recirculation matrix and HOM orientation, BBU can be controlled or alleviated altogether. The idea of being able to suppress BBU in the design and fabrication of RF cavities by a judicious orientation of the HOMs presents an appealing solution to alternative methods of BBU suppression requiring rotating optics schemes or external beam-based feedback. One could imagine fabricating a cavity in which the HOM coupler introduces the largest perturbations and so dictates the HOM polarization. In this way, one could orient the HOMs at will simply by choosing how to place the coupler.
Appendix A: Detailed Description of BBU Simulation Code

Here we describe a newly developed BBU simulation code (yet to be named). The primary reasons for writing a new code instead of using or modifying the existing codes TDBBU [8] and MATBBU [9] include:

- Slow execution speed of TDBBU and MATBBU
- Inconvenient interface for TDBBU and MATBBU
- A desire to include a feedback for BBU suppression
- Necessity of a correct treatment of the 2D transverse beam dynamics. Several attempts to run TDBBU and MATBBU with rotated HOMs and 4x4 matrices failed to produce meaningful results
- MATBBU and TDBBU are written in FORTRAN which makes their support under the Windows operating system difficult and expensive
- Modifying TDBBU and MATBBU and/or rewriting the codes in C/C++ would be more complicated and time consuming than writing a new code.

In the present configuration, the new code simulates beam dynamics in a two-pass machine. It assumes that bunches are point-like particles. In the linac of a two-pass recirculating machine, bunches that pass through the linac for the first time are interspaced with recirculated bunches. Time intervals between bunches depend on the bunch repetition frequency and the recirculation time. The code assumes that the distance between bunches does not change in the linac. The tracking algorithm has four main steps:

1. An injected bunch propagates through the entire linac for the first time. During its passage, the bunch both excites voltage of HOMs and gets deflected by HOMs excited by previous bunches. After its passage, the bunch is stored in an array. The array contains all bunches present in the linac on the first pass and in the recirculation pass. The number of array elements is given by \( n = \text{int}(T_r f_b) + 1 \). In this formula, \( T_r \) is the recirculation time and \( f_b \) is the bunch repetition frequency.
2. The code updates the voltage of all HOMs to the time of arrival of the recirculated bunch that immediately follows the injected bunch.
3. The code extracts the last recirculated bunch from the array, multiplies the bunch coordinates by the recirculation matrix, and runs the bunch through the entire linac.
4. The code updates the voltage of all HOMs in the linac to the moment of arrival of the next injected bunch.

Steps 1-4 are repeated until the simulation time exceeds the run time specified in the input file.
One of the major advantages of this new code is the ability for each HOM mode to be oriented at an arbitrary angle. The HOM voltage has two components: real and imaginary. A bunch excites the real part when it passes through a cavity according to

\[ V_r = q c \left( \frac{\omega}{c} \right)^2 \left( \frac{R}{Q} \right) (x \cos(\alpha) + y \sin(\alpha)) \]

where \( \alpha \) is the mode rotation angle. The real part of the voltage is proportional to the electric field of the HOMs. The imaginary part of the voltage is proportional to the magnetic field of HOMs. The imaginary part of the voltage deflects bunches according to

\[ \delta x' = \frac{V_i \cos(\alpha)}{V_b} \]
\[ \delta y' = \frac{V_i \sin(\alpha)}{V_b} \]

where \( V_b \) is the beam voltage. To calculate evolution of the HOM voltage in time, the code uses the following transformation

\[
\begin{bmatrix}
V_r \\
V_i
\end{bmatrix}_{t+dt}
= e^{-\frac{\omega dt}{2Q}}
\begin{bmatrix}
\cos(\omega dt) & -\sin(\omega dt) \\
\sin(\omega dt) & \cos(\omega dt)
\end{bmatrix}
\begin{bmatrix}
V_r \\
V_i
\end{bmatrix}_t,
\]

where \( \omega \) and \( Q \) are the frequency and the quality factor of the HOM, respectively, and \( dt \) corresponds to either the time interval between an injected bunch and a following recirculated bunch or a time interval between a recirculated bunch and a following injected bunch. The transformation is equivalent to a rotation of a complex vector with exponentially decaying amplitude.

The code uses transfer matrices to calculate transverse coordinates of each bunch. In the current version of the code, the user has to provide all matrices. The matrices can be given in either \((x,x')\) or \((x,p_x)\) coordinate systems. In a 1D case, the user has to provide only 2x2 matrices. In a 2D case, all matrices have to be 4x4. The current version of the code does not include an automatic threshold-hunting routine. The code outputs bunch coordinates at the end of the linac after the second pass to a display screen. The user has to redirect the output to a file and judge the beam stability visually, using a graphics program.

The first version of the code has been developed and is being validated. The code is written in the Standard ANSI C++. It took approximately a month to develop and debug (in the “first approximation”) the code. The code has been used to simulate beam breakup in several 1D cases including the JLab FEL Upgrade. The results of simulations were in a 3% agreement with results simulated by TDBBU and MATBBU. For 1D cases, the new code gave results almost identical to those generated by the code \texttt{b1} [10], developed by I.
Bazarov of Cornell. The authors could not use TDBBU, MATBBU, or bi to simulate full, 2D transverse beam dynamics. However, the results of simulations calculated by the new code were compared to the formula presented in Reference [1]. The simulation results for 2D cases presented in this paper agree perfectly with the theory. The new code is faster than TDBBU and MATBBU by an order of magnitude or more depending on the particular problem, and more than a factor of 3 faster than bi.

A “roadmap” of the foreseeable upgrades to this code is as follows:

- Upgrade of the code output. After the upgrade, the code will be able to output the voltage of chosen HOMs and bunch coordinates after a specified cavity.
- A separate Java code will provide a user-interface (UI) for the existing C++ code. The interface for the first version of the Java code will be text based. The text based interface will be a prototype for a graphical user-interface (GUI). The platform independence of Java will simplify the transfer of the code between platforms after the interface becomes graphic. In addition to taking care of basic interface responsibilities, the Java code will be able to run the C++ code in three different regimes:
  1. Single run
  2. Threshold current hunting, multiple runs
  3. Frequency scan, multiple runs

By implementing a separate Java interface, we achieve several goals:

  1. Implementation of current-hunting and frequency scanning subroutines
  2. The user still will be able to run the C++ code without using the interface
  3. The interface is platform independent.

- Implementation of a feedback to simulate beam-based BBU suppression methods
- Implementation of a Java GUI, based on the Java text UI as described above
- Upgrade the code to include multiple (more than two) pass machines.
References


(MATBBU was originally developed by B. Yunn: Yunn@jlab.org )

(see www.lns.cornell.edu/~ib38/bbocode/doc/bbudoc.pdf)