Simulating the Effects of a Bunch-by-Bunch Feedback System to Suppress Multipass Beam Breakup in an ERL

C. Tennant

Abstract

A simple beam breakup (BBU) code has been developed to study the effects of implementing a bunch-by-bunch feedback system on the threshold current. The code is used to validate an analytic model that describes the effects of feedback on BBU thresholds. However, the analytic model provides no insight into the behavior of the system in the pseudo-stable regime and simulation studies must be performed. Specifically, the effects of feedback time delays were investigated. Results indicate that, in principle, implementing a bunch-by-bunch feedback system in an ERL is feasible.

Introduction

Conceptually, a transverse feedback system is simple. A sensor or pickup is used to measure the beam displacement at a location in the machine. A kicker is located downstream and imparts an impulsive angular kick to the beam proportional to the offset signal detected at the pickup. The correcting kick may be applied on the same turn or on subsequent revolutions, it may be applied to the same bunch that produced the signal at the pickup or it may act on preceding or following bunches. All of these considerations are contingent upon the type of machine you are dealing with, the type and nature of the beam instability you wish to control and the signal processing required of the system, among other things.

Feedback systems have long been successfully used in storage rings. However, there are fundamental differences in designing a system for an energy recovery linac (ERL), namely the fact that the beam spends a relatively short time in the machine. Ideally one would like the feedback system to correct the same bunch that produced the error signal, but because in an ERL the beam makes only two passes through the machine, this requires that the bunch be corrected on the same turn on which the signal was detected. This imposes stringent requirements on the signal processing time and instrumentation electronics, which hitherto, had been thought to make such a system technically unfeasible [1].

Using an analytic model of BBU which incorporates the effects of a feedback system and using the results of a recently developed BBU code to simulate the effects of feedback on the system’s stability, this note describes how a bunch-by-bunch system may be feasible after all.
Analytic Model

An analytic model of BBU arising from a single HOM (assumed to be vertically polarized) in a single cavity with a single recirculation is well developed and has been validated using experimental data at the FEL Upgrade Driver [2]. In 2003, B. Yunn included the effects of a generic feedback system and derived the threshold current for stability [3]

\[ I_{th} = -\frac{2V_b e^{-\phi T_r}}{(R/Q) \omega T_{eff} \sin(\omega T_r + \phi)} \]  

where \( T_{eff} \) and \( \phi \) are defined from

\[ T_{eff} e^{-i\theta} = M_{34} + gM_{34}^pM_{34}^k e^{i\omega T_d} \]

The quantities in Eq. (1) are defined as follows: \( V_b \) is the beam voltage at the cavity, the dipole HOM is characterized by its angular frequency \( \omega \), loaded quality factor \( Q \) and the ratio \( R/Q \), \( k = \omega/c \) is the wavenumber, \( T_r \) is the recirculation time, \( T_d \) is the feedback delay time, \( g \) is the gain of the system, \( M_{34} \) is the recirculation element from the cavity back to itself, \( M_{34}^p \) transforms a kick from the cavity to a displacement at the location of the pickup and \( M_{34}^k \) transforms a kick at the location of the kicker to displacement at the cavity on the second pass.

Some time later an independent derivation of the threshold current incorporating feedback, and based on the formalism used in the theory of control systems, was completed by E. Pozdeyev. The result is [4]

\[ I_{th} = -\frac{2V_b}{(R/Q) \omega M_{eff}} \]  

where

\[ M_{eff} = M_{34} e^{\omega T_r/2Q} \sin(\omega T_r) + gM_{34}^pM_{34}^k e^{\omega (T_r + T_d)/2Q} \sin(\omega (T_r + T_d)) \]

Perhaps it is not immediately apparent, but Eqs. (1) and (2) are identical. The remainder of this note will use the nomenclature of Eq. (2).

For a physically viable solution, the threshold current must be a positive quantity. This condition requires that \( M_{eff} < 0 \) for Eq. (2) to be valid. Equations (1) and (2) are based on perturbative treatment of the problem. Thus, for the perturbative
solutions to be valid, the gain, $g$, must be less than $\left| M_{34} / M_{34}^p M_{34}^k \right|$ for $
abla \sin(\omega (T_r + T_d)) > 0$ and greater than $\left| M_{34} / M_{34}^p M_{34}^k \right|$ for $
abla \sin(\omega (T_r + T_d)) < 0$.

When these conditions are not met, the system is said to be in a pseudo-stable regime, where the negative threshold current implies beam stability. Recently, the behavior in the pseudo-stable regime has been investigated with numerical methods for the case of no feedback. The results show that beam breakup can still develop, but does so at currents of several Amperes [5]. For the 10 mA FEL Upgrade, and even for the more ambitious 100 mA ERL-based drivers, this represents, for all practical purposes, a stable system.

**Motivation**

The ultimate goal of the feedback system is to put the system in the pseudo-stable regime, effectively pushing the threshold current to several Amperes. While the analytic models provide insights into the behavior of the system in the regime where $M_{eff} < 0$, the region of greatest interest is the pseudo-stable regime, for which the analytic model can offer no information. Therefore it is necessary to investigate this regime with numerical methods using computer simulation codes.

Initial studies were performed by B. Yunn, who modified the BBU code TDBBU to include a simple feedback system for the case of $T_d = 0$. Results from those simulations indicated that an unstable system could be made stable by implementing such a system [3].

**Simulation Code**

A code to simulate beam dynamics in a two-pass machine for a cavity containing a single HOM which is assumed to be oriented either purely horizontally or vertically was developed. The code was written using Igor Pro [6] so that generating input files, executing the code and post-run analysis could be performed with the same program.

**Tracking Algorithm**

The tracking algorithm has the following steps:

1. The initially empty machine is filled with $(P_L / h) + 1$ bunches (truncated to the nearest integer) where $P_L$ is the recirculation path length in terms of RF wavelengths and $h$ is the beam repetition subharmonic. For the typical 74.85 MHz repetition rate used in the FEL Upgrade and with the 1497 MHz CEBAF cavities, $h = 20$.
2. An injected bunch propagates through the entire linac for the first time. Bunches up to a specified time are given initial displacement and/or an initial angle so as to excite the HOM. During its passage, the bunch excites the HOM voltage according to
where $V_R$ is the real component of the HOM voltage and $y_{c,1}$ is the first pass displacement of the bunch through the cavity. The code follows the convention that the real component corresponds to the electric field and is the means by which the beam bunch couples to the HOM.

3. The bunch is deflected by the HOM excited by the passage of previous bunches

$$y'_{c,1} = \frac{V_I}{V_b}$$

where $V_I$ is the imaginary component of the voltage and corresponds to the magnetic field which imparts an angular kick on the beam bunch. After its passage, the bunch is stored in an array. The array contains all bunches present in the linac on the first pass and in the recirculation pass.

4. Before the arrival of a recirculated bunch, the HOM voltage decays according to

$$\begin{bmatrix} V_R \\ V_I \end{bmatrix} = e^{\frac{-\omega dt}{2Q}} \begin{bmatrix} \cos(\omega dt) & -\sin(\omega dt) \\ \sin(\omega dt) & \cos(\omega dt) \end{bmatrix} \begin{bmatrix} V_R \\ V_I \end{bmatrix}$$

where $dt$ is the time interval between an injected bunch into the linac and a recirculated bunch.

5. The first pass beam is propagated from the cavity to a downstream pickup according to a user-input transfer matrix. The beam displacement at the pickup, which is used as the error signal to drive the feedback system’s kicker, is stored in an array.

6. The bunch is then transported to the kicker according to a user-input transfer matrix. The kicker imparts a transverse deflection which is proportional to the displacement at the pickup with a gain set by the user. For a feedback time delay of zero, the kicker simply imparts a kick to the bunch in proportion to the displacement of that same bunch at the pickup. The code also handles the more interesting case involving nonzero feedback time delays. In these instances the beam displacement is stored in an array and used at the kicker only after the passage of $T_d \left( f_{RF} / h \right)$ bunches, where $T_d$ is the specified feedback time delay and $f_{RF} / h$ is the bunch repetition frequency.

7. The beam bunch is transported from the kicker to the cavity according to a user-input transfer matrix.

8. The second pass beam bunch then induces a voltage in the cavity according to Eq. (3) where $y_{c,1}$ is replaced by the second pass displacement.
9. The bunch coordinates and the real and imaginary HOM voltages are written to a data file for post-run analysis.

10. The cavity voltage is allowed to decay according to Eq. (5) where $dt$ is the time interval between the recirculated bunch and the next injected bunch.

Steps 2-10 are repeated until the simulation time exceeds the specified run time. The above algorithm, save for steps 4-6, are the basis of any BBU simulation code based on the method of particle tracking. Such codes include TDBBU, ERLBBU, GBBU and BI. The simulations also utilize a binary search algorithm to automatically search for the threshold current.

Because it is assumed that the pickup can generate a position signal for each beam bunch and likewise, that the kicker can impulsively kick each bunch independently, this represents an idealized model. In reality, the signal produced by a single bunch through a pickup-amplifier-kicker system will affect more than a single bunch [7]. This model does not take these effects into account, nevertheless, important insights can be gained about the performance of a feedback system.

The reason for creating a code capable (at this point) of modeling only a single mode is due to the fact that, to a high degree of accuracy, BBU in the FEL Upgrade Driver can be described with the single mode analytic formula, i.e. Eq. (1) or (2) with $g = 0$. (See for example References [2] and [5] for a more thorough discussion on the topic.) By simulating even a single mode, important insights into the behavior of a feedback system can be gained.

**Benchmarking the Code**

To make certain that the simulation code was working correctly, it was benchmarked with the results of ERLBBU for the case of no feedback, that is, with $g = 0$ in Eq. (2). The two codes show perfect agreement. Figure 1 shows the output values of the imaginary voltage from ERLBBU and from the new code during a typical run.
The next step requires benchmarking the code with the analytic models for the case of bunch-by-bunch feedback with $T_d = 0$. The results of simulations for the remainder of this note use the following input parameters:

\[ f = 2106.007 \text{ MHz} \]
\[ Q = 6.11 \times 10^6 \]
\[ R/Q = 29.9 \Omega \]
\[ V_b = 39 \text{ MV} \]
\[ T_r = 433.200 \text{ ns} \]
\[ M_{34} = -5.2 \text{ m} \]
\[ M_{34}^p = 1.1 \text{ m} \]
\[ M_{34}^k = 13.7 \text{ m} \]

where the matrix elements were extracted from all-save values from January 18, 2005 of an 88 MeV setup. The value for $M_{34}$ is from zone 3 cavity 7 (the location of the unstable mode) back to itself, the value for $M_{34}^p$ is for a pickup located in the 2F region and the value for $M_{34}^k$ reflects a kicker located in the 5F region. For the model to be valid, recall that $g < \left| M_{34} / M_{34}^p M_{34}^k \right| (= 0.35)$.

The agreement between the analytic formula and the results of the simulation are summarized in Fig. 2. Without feedback, the threshold current is 2.1 mA. It is clear that the analytic model is correct in its region of validity (or that the simulation code is correct, depending on your point of view). The more interesting situations,
however, are for time delays in the feedback system \( T_d \neq 0 \) and the behavior of the system in the pseudo-stable regime \( M^{\text{eff}} > 0 \).

![Graph showing the threshold current as a function of gain for \( T_d = 0 \) from the analytic model (red line) the results of the simulation code (black open circles).](image)

**Bunch-by-Bunch Feedback**

Initially, the challenges of implementing a bunch-by-bunch feedback system in a relatively small, two pass ERL such as the FEL Upgrade Driver seemed prohibitive (at least to the author). The primary obstacle is the time budget and the effects of not meeting that budget.

Consider the feedback time budget. The recirculation time in the FEL Upgrade Driver is 433.200 ns. In terms of time management, the ideal placement of the pickup is immediately downstream of zone 4 and the ideal placement of the kicker is immediately upstream of zone 2. With this configuration the distance between the pickup and kicker is approximately 50 m. For optimal feedback performance, for each bunch that gets a correcting kick, the error signal used is the one generated by that same bunch. That is, the condition where \( T_d = 0 \). Yet, after taking into account the propagation time for a signal from the pickup to reach the kicker, only a few tens of nanoseconds remain in the time budget for processing the raw BPM signal, generating a suitable error signal and supplying sufficient gain. It was therefore not immediately clear whether adequate suppression could be achieved for the case where \( T_d \neq 0 \). As a result, other feedback mechanisms were proposed in lieu of a bunch-by-bunch system [1]. However, as simulation results presented in the following section indicate, a bunch-by-bunch feedback system may be more realistic than previously thought.
Simulation Results

A number of simulations were performed to ascertain some of the parametric dependencies in the pseudo-stable regime. All simulations were performed with the parameters listed earlier. The remaining free parameters are the feedback time delay ($T_d$) and the feedback gain ($g$).

As discussed in the previous section, the effect of feedback time delay is the most important issue for determining if sufficient suppression can be achieved if the bunch corrected/kicked uses an error signal derived from a different bunch. Simulation results showing the dependence of the threshold current on the time delay (for $g = 1$) are given in Fig. 3. The value of threshold current oscillates and as the time delay increases, the maximum achievable threshold decreases according to a power law. The functional dependence of the maximally achieved threshold current as a function of the time delay is shown in the log-log plot in Fig. 4. The data is fit with a straight line of slope $-0.93$. Thus, for the parameters used in these studies, the maximum threshold possible by implementing a bunch-by-bunch feedback system scales as $T_d^{-0.93}$.

![Threshold Current versus the Feedback Time Delay](image)

Fig. 3: Threshold current versus the feedback time delay. As the time delay gets longer the maximum achievable threshold decreases according to a power law (see Fig. 4).
Figure 4: The maximum threshold current that can be achieved with feedback as a function of time delay. The best fit line has a slope of $-0.93$.

Figure 5 shows the threshold current as a function of the feedback gain for several different values of time delay. The values of $T_d$ were chosen such that they correspond to the maximum achievable threshold current (i.e. the peaks in Fig. 3). In the region for which the analytic model is valid ($g < 0.35$) the simulations show excellent agreement, save for the case of the longest time delay (423 $\mu$s) where the perturbative treatment of the problem begins to fail.

Fig. 5: The threshold current as a function of feedback gain for several different time delays. The lower and upper dotted lines mark threshold currents of 2.1 mA and 21 mA, respectively.
From a practical point of view, for $T_d < 423 \, \mu s$ and for $g = 1$, the threshold current can be increased with a feedback system. Not surprisingly, the best suppression occurs when $T_d = 0$ and the feedback is truly on a bunch by bunch basis. To achieve an order of magnitude increase in the threshold current, from 2.1 mA to 21 mA, requires a feedback time delay of less than 30 $\mu s$. Conversely, for delays greater than 423 $\mu s$, the threshold current becomes completely ineffective, independent of the gain.

Finally, the relationship between the required feedback time delay to achieve a threshold current of 2.1 mA as a function of the gain was investigated. The resulting isoline is given on the log-log plot in Fig. 6. The data is fit with a straight line of slope $-0.86$. Thus, for the parameters used in these studies and for a 2.1 mA threshold current, the feedback time delay scales as $g^{-0.86}$. For each value of the gain, time delays which lie below the best fit line will lead to a threshold current which exceeds 2.1 mA while points above lead to thresholds less than 2.1 mA.

**Fig. 6:** The feedback time delay as a function of gain. The best fit line has a slope of $-0.86$.

**Conclusions and Implications**

A simple BBU code has been developed to study, in particular, the effects of time delays for implementing a bunch-by-bunch feedback system. While the details of the results reported in the previous section depend largely on the specific choice of simulation parameters, in general, the following conclusions can be drawn

1. The most effective suppression for BBU occurs when $T_d = 0$, although effective suppression of BBU can be arranged for finite feedback time delays
2. The maximum threshold current that can be obtained by implementing feedback decreases as the time delay increases (the rate at which this occurs depends on the simulation input parameters).

3. For large time delays, a feedback system will only decrease the threshold – regardless of the gain.

The last point is worth emphasizing; unless the required time budget can be met, the feedback will be completely ineffective. In fact, using the feedback in this regime will act to only decrease the threshold current further.

The purpose of the study and this note is to point out that, in principle, implementing a bunch-by-bunch feedback system in an ERL, specifically the FEL Upgrade Driver, is feasible. However, technical issues regarding the hardware have, as of yet, not been addressed (e.g. the placement of pickups and kickers, BPM resolution, kicker power and other similar issues).

References


