A STUDY OF BEAM BREAKUP IN 12 GeV UPGRADE WITH DOUBLE BEND ACHROMAT ARC OPTICS
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1. INTRODUCTION
Previously, an HOM Damping Requirement Study for the 12 GeV Upgrade (JLAB-TN-04-035) was done using a beam transport design similar to the present 4 GeV standard arc optics. However, when the present arc optics are used for the 12 GeV upgrade, the emittance and energy spread increase very much due to synchrotron radiation in the arcs. As an alternative proposal, Double Bend Achromat Arc Optics for 12 GeV CEBAF (JLAB-TN-07-010) were developed by Alex Bogacz. The main purpose of this study is to compare the threshold current for two optics, and the BBU likelihood of 7-cell cryomodules in terms of location is also examined.

2. DBA ARC OPTICS
In this document the arcs are measured from the center of the quad immediately after the exit of the last cryomodule of a linac to the center of the quad immediately before the first cryomodule of the next linac. X and Y are respective horizontal and vertical coordinates in cm unit. PX and PY are respective momentum in MeV/c unit.

Arc 1:
\[
\begin{align*}
X &= 0.473460E-01 \times X + 0.240458E+00 \times PX \\
PX &= -0.414948E+01 \times X + 0.469064E-01 \times PX \\
Y &= 0.971967E+00 \times Y - 0.947783E+00 \times PY \\
PY &= 0.852117E-01 \times Y - 0.945750E+00 \times PY
\end{align*}
\]

Arc 2:
\[
\begin{align*}
X &= 0.782088E+00 \times X + 0.716039E+00 \times PX \\
PX &= 0.358737E+00 \times X + 0.950187E+00 \times PX \\
Y &= 0.167725E+01 \times Y + 0.687543E+00 \times PY \\
PY &= 0.112328E+01 \times Y + 0.135758E+00 \times PY
\end{align*}
\]

Arc 3:
\[
\begin{align*}
X &= 0.499778E+00 \times X + 0.112934E+01 \times PX \\
PX &= 0.778055E+00 \times X + 0.242729E+00 \times PX \\
Y &= 0.705404E+00 \times Y + 0.905889E+00 \times PY \\
PY &= 0.492629E+00 \times Y + 0.784986E+00 \times PY
\end{align*}
\]

Arc 4:
\[
\begin{align*}
X &= 0.176843E+01 \times X + 0.493148E+00 \times PX \\
PX &= -0.428093E+00 \times X + 0.446096E+00 \times PX \\
Y &= -0.170947E+01 \times Y + 0.767124E+00 \times PY \\
PY &= 0.108782E+01 \times Y + 0.968173E-01 \times PY
\end{align*}
\]
Arc 5:
X = 0.742225E+00*X + 0.813432E+00*PX
PX = -0.133096E+01*X + -0.111347E+00*PX
Y = -0.111790E+01*Y + -0.940154E+00*PY
PY = 0.616815E+00*Y + -0.375793E+00*PY

Arc 6:
X = 0.881379E-01*X + -0.106344E+01*PX
PX = 0.922957E+00*X + 0.209792E+00*PX
Y = -0.264600E+01*Y + -0.239376E+00*PY
PY = 0.652805E+00*Y + -0.318872E+00*PY

Arc 7:
X = -0.439817E+00*X + -0.122282E+01*PX
PX = 0.107457E+01*X + 0.713945E+00*PX
Y = -0.395057E+00*Y + 0.707152E+00*PY
PY = -0.710459E+00*Y + -0.125956E+01*PY

Arc 8:
X = -0.154373E+01*X + 0.830844E-01*PX
PX = 0.412754E+00*X + -0.669997E+00*PX
Y = -0.805295E+00*Y + 0.936958E+00*PY
PY = -0.718766E+00*Y + -0.405499E+00*PY

Arc 9:
X = -0.566666E+00*X + -0.412663E+00*PX
PX = 0.128165E+00*X + -0.167137E+01*PX
Y = 0.721146E+00*Y + 0.493421E+00*PY
PY = -0.272457E+00*Y + 0.120026E+01*PY

3. BEAM BREAKUP SIMULATIONS

Several assumptions and approximations have been utilized in order to make the calculations more efficient. These approximations should still be closely representative of a machine which is dominated by a single HOM in the cryomodules.

- Two HOMs, 1874 MHz and 2111 MHz, are considered.
- Only one mode is excited in each cavity.
- 7-cell cavity cryomodules are located at the 21st ~ 25th slots in the North linac and 46th ~ 50th slots in the South linac.
- Only certain cavities are exited with an HOM in each threshold calculation. The other cavities give energy gains without excitation of HOMs.
- The total recirculation path length is 6554(6549, 6547, 6546) RF wavelengths for the 1st (2nd, 3rd, 4th) pass of the CEBAF accelerator.
3.1. Threshold current using DBA optics

The same sample number (500), HOM (1874 MHz), and Q value ($10^7$) are used as in the figure 3 in JLAB-TN-035 in order to compare threshold current of previous 12 GeV optics with DBA optics. All 7-cell cavities in 10 cryomodules are excited at the same time. Threshold current for DBA optics is 0.219 mA, compared to 0.231 mA previously. Threshold current decreases by approximately 5.2%.

![Diagram](image)

Table 1. BBU threshold distribution for the 1874 MHz mode with Q=$10^7$ using DBA optics. All 7-cell cavities are excited simultaneously. 500 samples, Minimum $I_{th}$ = 0.219 mA

3.2. Dependence of threshold current on Q

Cavities located in the 21st cryomodule are excited with only 1874 MHz mode, and every Q is varied from $10^3$ to $10^8$. When Q > $10^6$, the BBU threshold current is inversely proportional to Q. Therefore, we can safely obtain threshold current for Q>$10^6$ by scaling with Q values. Threshold current 0.219 mA for Q=$10^7$ can be scaled down to 2.19 mA for Q=$10^6$.

<table>
<thead>
<tr>
<th>Q</th>
<th>Threshold current</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>1454.53 mA</td>
</tr>
<tr>
<td>$10^4$</td>
<td>268.3 mA</td>
</tr>
<tr>
<td>$10^5$</td>
<td>50.5 mA</td>
</tr>
<tr>
<td>$10^6$</td>
<td>5.7 mA</td>
</tr>
<tr>
<td>$10^7$</td>
<td>0.57 mA</td>
</tr>
<tr>
<td>$10^8$</td>
<td>0.057 mA</td>
</tr>
</tbody>
</table>

Table 2. Dependence of threshold current on Q
3.3. Threshold distribution changing the location of excited cryomodule

3.3.1. The most and the least likely place for BBU

The 21\textsuperscript{st} cryomodule location was thought to be most unstable place for BBU due to the lowest beam energy, and the 50\textsuperscript{th} cryomodule location was expected as the safest location on account of the highest beam energy. For the purpose of confirming this, the threshold currents were obtained for $Q=10^6$ as the following table shows. Only the 21\textsuperscript{st} or 50\textsuperscript{th} cryomodule is excited with an HOM.

<table>
<thead>
<tr>
<th>Excited cryomodule</th>
<th>21\textsuperscript{st}</th>
<th>50\textsuperscript{th}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1874 MHz</td>
<td>4.792 mA</td>
<td>22.05 mA</td>
</tr>
<tr>
<td>2111 MHz</td>
<td>4.763 mA</td>
<td>22.07 mA</td>
</tr>
</tbody>
</table>

Table 3. The threshold current when the 21\textsuperscript{st} or 50\textsuperscript{th} cryomodule is excited.

When the 21\textsuperscript{st} cryomodule is excited the threshold current is about 4.8 mA, but when every cryomodule is excited at the same time, the actual threshold current is quite different value; 2.19 mA for $Q=10^6$. Therefore it is believed that the other cryomodule plays the main role in BBU. In the next section, each cryomodule is excited one by one in order to identify the location of the most dangerous cryomodule for BBU.

3.3.2. Finding the location of the most dangerous cryomodule for BBU

For the purpose of investigating the effects each cryomodule has on BBU, the cavities in only one cryomodule at a time are excited, one after the other. Threshold currents are calculated when $Q=10^6$ and just one HOM is excited. According to the data in the following tables, the threshold currents are minimums when the cavities in the 25\textsuperscript{th} cryomodule are excited with 1874 MHz or 2111 MHz, and therefore the 25\textsuperscript{th} cryomodule is the most likely candidate for BBU. By comparing threshold currents in the table 1 and 2, BBU is less likely to occur in the south linac than in the north.

<table>
<thead>
<tr>
<th>Excited cryomodule</th>
<th>21\textsuperscript{st}</th>
<th>22\textsuperscript{nd}</th>
<th>23\textsuperscript{rd}</th>
<th>24\textsuperscript{th}</th>
<th>25\textsuperscript{th}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1874 MHz</td>
<td>4.792 mA</td>
<td>2.81 mA</td>
<td>2.31 mA</td>
<td>2.31 mA</td>
<td>1.97 mA</td>
</tr>
<tr>
<td>2111 MHz</td>
<td>4.763 mA</td>
<td>2.79 mA</td>
<td>2.59 mA</td>
<td>2.33 mA</td>
<td>1.96 mA</td>
</tr>
</tbody>
</table>

Table 4. North linac cavities excited

<table>
<thead>
<tr>
<th>Excited cryomodule</th>
<th>46\textsuperscript{th}</th>
<th>47\textsuperscript{th}</th>
<th>48\textsuperscript{th}</th>
<th>49\textsuperscript{th}</th>
<th>50\textsuperscript{th}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1874 MHz</td>
<td>11.95 mA</td>
<td>8.82 mA</td>
<td>27.84 mA</td>
<td>19.20 mA</td>
<td>22.05 mA</td>
</tr>
<tr>
<td>2111 MHz</td>
<td>11.87 mA</td>
<td>8.72 mA</td>
<td>27.31 mA</td>
<td>19.67 mA</td>
<td>22.07 mA</td>
</tr>
</tbody>
</table>

Table 5. South linac cavites excited

Threshold current distributions for each cryomodule excitation are in the following sections.
3.3.2.1. The cavities with \( Q=10^6 \) are located in the 21\textsuperscript{st}-25\textsuperscript{th} and 46\textsuperscript{th}-50\textsuperscript{th} cryomodule. HOM frequencies are distributed randomly around 1874 MHz with the full width of 5 MHz in the cavities. The following histograms show the threshold current distributions using 200 samples when each cryomodule is excited respectively.

Figure 1. HOMs around 1874MHz are excited in 21\textsuperscript{st} cryomodule cavities with \( Q=10^6 \) and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)

Figure 2. HOMs around 1874MHz are excited in 22\textsuperscript{nd} cryomodule cavities with \( Q=10^6 \) and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)
Figure 3. HOMs around 1874MHz are excited in 23\textsuperscript{rd} cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)

Figure 4. HOMs around 1874MHz are excited in 24\textsuperscript{th} cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)
Figure 5. HOMs around 1874MHz are excited in 25th cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)

Figure 6. HOMs around 1874MHz are excited in 46th cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)
Figure 7. HOMs around 1874MHz are excited in 47th cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)

Figure 8. HOMs around 1874MHz are excited in 48th cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)
Figure 9. HOMs around 1874MHz are excited in 49\textsuperscript{th} cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)

Figure 10. HOMs around 1874MHz are excited in 50\textsuperscript{th} cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)
3.3.2.2. 2111 MHz mode is excited with the full width of 5 MHz with $Q=10^6$ in the cavities located in the 21\textsuperscript{st}-25\textsuperscript{th} and 46\textsuperscript{th}-50\textsuperscript{th} cryomodule. The following histograms show the threshold current distributions using 200 samples when each cryomodules are excited respectively.

Figure 11. HOMs around 2111 MHz are excited in 21\textsuperscript{th} cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)

Figure 12. HOMs around 2111 MHz are excited in 22\textsuperscript{nd} cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)
Figure 13. HOMs around 2111 MHz are excited in 23\textsuperscript{th} cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)

Figure 14. HOMs around 2111 MHz are excited in 24\textsuperscript{th} cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)
Figure 15. HOMs around 2111 MHz are excited in 25th cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)

Figure 16. HOMs around 2111 MHz are excited in 46th cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)
Figure 17. HOMs around 2111 MHz are excited in 47th cryomodule cavities with Q=10^6 and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)

Figure 18. HOMs around 2111 MHz are excited in 48th cryomodule cavities with Q=10^6 and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)
Figure 19. HOMs around 2111 MHz are excited in 49\textsuperscript{th} cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)

Figure 20. HOMs around 2111 MHz are excited in 50\textsuperscript{th} cryomodule cavities with $Q=10^6$ and the frequencies are distributed randomly with the full width of 5 MHz. (200 samples)
3.4. Summing up the results of each cryomodule excitation

Every result in section 3.3 is put together in one histogram to see the overall distributions. Figure 21 shows the overall threshold distribution for 1874 MHz mode and figure 22 zooms in on 0–100 mA. These statements also apply to figures 23 and 24, but for the 2111 MHz mode.

Figure 21. All data for 1874 MHz are put together in one histogram. (2000 samples)

Figure 22. All data for 1874 MHz are put together in one histogram. Cut from 0 to 100 in X axis. (2000 samples)
Figure 23. All data for 2111 MHz are put together in one histogram. (2000 samples)

Figure 24. All data for 2111 MHz are put together in one histogram. Cut from 0 to 100 in X axis. (2000 samples)
3.5. Sensitivity of threshold current to the change of HOM frequency

To examine how sensitive the threshold current is to HOM change, the threshold current is calculated as the HOM frequency is varied from 1874 MHz to 1885 MHz. Four cases are considered for this work. In two cases only one cryounit is excited with $Q=10^6$, and in the other two cases four cryounits, three with $Q=10^3$ and one with $10^6$, are excited. Through this, we have found the following:

- As HOM frequencies vary, the threshold currents undergo a rapid change. More than 4 periods happens within a 1 MHz interval as the figure 25~28 show.

- A 0.1 MHz change in HOM frequency gives several tens of magnitude difference in threshold current as in the figure 25~28.

- If we took a small number of samples for HOM frequencies, we would be unable to make accurate conclusions as to the threshold current.

- Comparing figures 25 and 26 or figures 27 and 28, we see that if one cavity’s $Q$ is much larger than the others, for example $10^3$ and $10^6$ as in the figures 26 and 28, the large $Q$’s cavity dominates BBU. Therefore we could safely ignore the low $Q$ cavities.

- Comparison of figure 25 with 27 or figure 26 with 28 shows that the different location of the cavity changes the patterns of threshold current, but the periods are almost the same.

The following four graphs explain the above facts. The 21st cryomodule is considered to make the graphs.
3.5.1. Only the 1\textsuperscript{st} cryounit with Q=10\textsuperscript{6} are exited. Minimum $I_{th}=4.9348$ mA, Maximum $I_{th}=143.582$ mA

![Figure 25. Only the 1\textsuperscript{st} cryounit is exited in the 21\textsuperscript{st} cryomodule and the HOM frequency is scanned between 1874 MHz and 1875 MHz. Minimum $I_{th}=4.9348$ mA, Maximum $I_{th}=143.582$ mA](image)

3.5.2. Only the 4\textsuperscript{th} cryounit with Q=10\textsuperscript{6} is exited. Minimum $I_{th}=4.9346$ mA, Maximum $I_{th}=143.574$ mA

![Figure 26. Only the 4\textsuperscript{th} cryounit is exited in the 21\textsuperscript{st} cryomodule and the HOM frequency is scanned between 1874 MHz and 1875 MHz. Minimum $I_{th}=4.9346$ mA, Maximum $I_{th}=143.574$ mA](image)
3.5.3. The 1st cryounit has $Q=10^6$ and the 2nd, 3rd, and 4th cryounits have $Q=10^3$. Threshold current is examined as the HOM frequency changes at the 1st cryounit from 1874 MHz to 1875 MHz. Minimum $I_{th}=5.5077 \text{ mA}$, Maximum $I_{th}=355.974 \text{ mA}$

Figure 27. $Q=10^6$ for the cavities at the 1st cryounit and $Q=10^3$ for the cavities at the 2nd, 3rd, and 4th cryounits. HOM frequency at the 1st cryounit is scanned from 1874 MHz to 1875 MHz. Minimum $I_{th}=5.5077 \text{ mA}$, Maximum $I_{th}=355.974 \text{ mA}$

3.5.4. The 1st, 2nd, and 3rd cryounits have $Q=10^3$ and the 4th cryounit have $Q=10^6$. Threshold current examined as HOM frequency changes at the 4th cryounit from 1874 MHz to 1875 MHz. Minimum $I_{th}=5.5076 \text{ mA}$, Maximum $I_{th}=355.8950 \text{ mA}$

Figure 28. $Q=10^3$ for the cavities at the 1st, 2nd, and 3rd cryounits and $Q=10^6$ for the cavities at the 4th cryounit. HOM frequency at the 4th cryounit is scanned between 1874 MHz and 1875 MHz. Minimum $I_{th}=5.5076 \text{ mA}$, Maximum $I_{th}=355.8950 \text{ mA}$
4. CONCLUSIONS

- A large number of samples are needed in the BBU study. Small numbers of HOM samples could give premature results because the threshold current changes very rapidly with respect to HOM frequency. The histograms with 200 samples do not show good statistical distributions. An order of magnitude more samples would be needed for good statistics.

- Regardless of statistical uncertainty, the threshold current for DBA optics is 2.19 mA when $Q=10^6$, while in the previous 12 GeV optics study (JLAB-TN-04-035) it was 2.31 mA. The threshold current decreases by about 5.2%.

- If we want the same threshold current as before, the previous $Q$ requirement should be lowered by 5.2%. However the previous HOM damping requirements had a safety margin which multiplied the value by 3. It means the threshold current was three times larger than the required value. Even though we consider a 5.2% reduction in threshold current, the safety margin is still 2.84. Therefore the previous damping requirement is safe and acceptable.

- When $Q > 10^6$, the BBU threshold current is inversely proportional to $Q$ values. Therefore we can safely obtain threshold current for $Q>10^6$ by scaling with $Q$ values.

- The $25^{th}$ cryomodule is the most likely place for BBU, and the $50^{th}$ the least place. The South linac is less likely to have BBU than the North linac. In the following table, the cryomodules are in order of likelihood of BBU.

<table>
<thead>
<tr>
<th>BBU likelihood</th>
<th>Likely ←----------------------------------------→ Unlikely</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cryomodule location</td>
<td>$25^{th}$ $24^{th}$ $23^{rd}$ $22^{nd}$ $21^{st}$ $47^{th}$ $46^{th}$ $49^{th}$ $48^{th}$ $50^{th}$</td>
</tr>
</tbody>
</table>

Table 6. Cryomodules in order of BBU likelihood

- The placement of cryomodule with the lowest $Q$ should be at the $25^{th}$ place, the most likely place for BBU, and the higher $Q$ value, the further to the right in the table above. The cryomodule with the highest $Q$ should be located at the $50^{th}$ place, the least likely place for BBU. In summary, the placement of cryomodules with respect to $Q$ values needs to follow the order in the table below.

<table>
<thead>
<tr>
<th>Q value</th>
<th>Lower Q ←----------------------------------------→ Higher Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cryomodule location</td>
<td>$25^{th}$ $24^{th}$ $23^{rd}$ $22^{nd}$ $21^{st}$ $47^{th}$ $46^{th}$ $49^{th}$ $48^{th}$ $50^{th}$</td>
</tr>
</tbody>
</table>

Table 7. Placement of cryomodule with respect to Q values
5. ACKNOWLEDGEMENT

Help from Yves Roblin and Ryan Bodenstein is greatly appreciated.