The Phase Noise and Jitter Requirements for the CEBAF Master Oscillator
Curt Hovater

Introduction
The CEBAF master oscillator (MO) or reference is being replaced with an updated system that
will make path length changes easier. For the new system we intend to use a commercial “off the
shelf” signal generator. When evaluating a master oscillator it is important to keep in mind that
the accelerators energy spread requirement is directly tied to the phase stability in the
accelerating cavities. This stability is partly defined by the spectral purity of the master oscillator
system. In CEBAF, the cavity phase stability requirement is 0.25° rms. (correlated) [1]. The
uncorrelated requirement is 0.5° rms. Given that between cavities the MO is a correlated noise
source, it makes sense to use the correlated requirement. To make this easier for comparison we
need to convert the phase requirement to a timing jitter. 0.25° converts to a timing jitter of 464 fs
at 1497 MHz. From this we can compare commercial (Agilent 4438A and Rhode Schwarz
SMA100A) signal generators phase noise to the CEBAF timing jitter requirement.

Phase Noise and Jitter
Signal generators signal purity is typically specified as a phase noise. Phase noise is defined as
the ratio of the noise in a 1-Hz bandwidth at a specified frequency offset from the oscillator signal
amplitude at frequency [2]. Typically this offset is between 10 Hz and 1 MHz. The phase noise is
expressed in dBc/Hz. dBc/Hz is the amount of spectral noise below the carrier (hence the c in
dBc) in a 1-Hz bandwidth (hence the Hz). Figure 1 shows the phase noise spectral density of
three signal sources (Agilent, Rhode Schwarz and Wenzel).

<table>
<thead>
<tr>
<th>Signal Source Phase Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agilent 850 MHz</td>
</tr>
<tr>
<td>Wenzel 70 MHz</td>
</tr>
<tr>
<td>RS 1GHz</td>
</tr>
</tbody>
</table>

![Figure 1. Phase noise plots of three different signal sources](image)
When looking at a signal generators specification you are given phase noise at only a couple of frequencies. To be useful we need to convert the phase noise spectrum to a timing jitter and adjust it to match our frequency. This can be done by integrating the area under the phase noise curve and converting it to radians and then to seconds

$$\tau = \sqrt{\frac{2 \cdot 10^{4/10}}{2\pi f_o}}$$  \hspace{1cm} [1]$$

where A is the area under the phase noise curve and fo is the signal source frequency. Using equation 1 and the information in figure 1 we can then calculate the rms. jitter for each source.

In this method we only calculate the integrated noise between 6 frequencies (10, 100, 1,000, 10,000, 100,000 and 1,000,000 Hz). To check the accuracy of such a method we measured the phase noise of an Agilent 4428B signal source at 850 MHz (Figure 2) and compared it to the integrated jitter calculated from the 6 frequencies. We used an Agilent Signal Source analyzer to measure the phase noise. The calculated method gave us 189 fs while the measured jitter was 206 fs. So using phase noise data supplied by the manufacture and integrating between six frequencies will give an underreporting of the jitter. In the case of the Agilent it is 16 fs. Since the curves are similar we should expect similar results for the Rhode Schwarz.

![Figure 2: Phase noise of Agilent 4428B at 850 MHz](image-url)
**CEBAF Master Oscillator and Distribution**

In CEBAF the MO distributes 70 MHz and 499 MHz to each service building from the Machine Control Center. In the service building the 499 MHz is multiplied by three to generate the RF cavity frequency of 1497 MHz. In addition a ‘Local oscillator” or LO signal is generated by mixing the RF signal with the 70 MHz or IF. This gives us 1427 MHz which is then amplified and distributed down the service building. The frequencies are recombined at the Low Level RF (LLRF) controllers to drive the superconducting cavities.

With that in mind we need to determine the jitter of the 1427 MHz and the 70 MHz at the LLRF controllers. From Figure 1 we can get an integrated phase noise of the commercial sources at their respective measured frequency. Next we need to adjust the phase noise to the cavity frequency of 1497 MHz. We can apply the simple formula $20\log_{10}(N)$, where $N$ is the multiplication factor to get to 1497 MHz. By adding this to the integrated noise we can get an integrated phase noise at 1497 MHz and the jitter. Table 1 shows the integrated phase noise and jitter projected at 1497 MHz, in addition for comparison the intermediate frequency (IF) source is given at 70 MHz.

<table>
<thead>
<tr>
<th>Signal Source</th>
<th>Integrated Phase Noise (dBc/MHz)</th>
<th>Jitter (RMS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agilent (1497 MHz)</td>
<td>-67.5</td>
<td>63 fs</td>
</tr>
<tr>
<td>Rhode Schwarz (1497 MHz)</td>
<td>-64.0</td>
<td>76 fs</td>
</tr>
<tr>
<td>Wenzel (70 MHz)</td>
<td>-97.1</td>
<td>45 fs</td>
</tr>
</tbody>
</table>

At first glance both signal sources, the Agilent and the Rhode Schwarz, compare favorably with the required jitter specification, 464 fs. To get the jitter specification of the local oscillator (LO) we must include the mixing effect with the 70 MHz IF.

A mixer is simply a multiplier and the two signals can be treated as such. In this case we have a RF and an IF signal which can be represented as

$$RF = \sin(\omega_{RF}t + \phi_{RF}(t))$$
$$IF = \sin(\omega_{LO}t + \phi_{LO}(t))$$

Where $\omega$ is the frequency and $\phi(t)$ is the signals phase noise. Multiplying the two signals together gives

$$LO = \frac{1}{2} \left( \cos[(\omega_{RF}t + \phi_{RF}(t)) - (\omega_{LO}t + \phi_{LO}(t))] + H.O.terms \right)$$

[2]

[3]
We see here the familiar frequency subtraction to get the LO and in this case we will ignore the higher order terms. The phase noise components do not subtract but are added together. Taking this into effect the LO will be the following

\[ LO = \frac{1}{2} \left( \cos \left( \omega_{RF} t + \phi_{RF}(t) + \phi_{LO}(t) \right) \right) \]  

[4]

The worst case would be where the two phase noises add together. In our case by adding the integrated phase noise of the RF and the IF phase together we then can calculate the LO jitter of 67 fs. The phase noise content of the IF is much smaller (-20 dB) than the RF sources, so the contribution is small (i.e. the LO is dominated by the 499 MHz signal source and not the 70 MHz source).

**Amplification Effects on Phase Noise**

To be complete we must include in contributions along the way due to amplification. Affects of amplification on phase noise are typically small. They can be estimated by adding a flicker noise \(1/f\) contribution to the spectral density along with the amplifiers noise figure

\[ S_\phi(f) = \frac{\alpha}{f} + \frac{2kTFG}{P} \]  

[5]

where \(\alpha\) is the flicker noise coefficient, \(k\) is Boltzmann’s constant, \(T\) is the temperature in Kelvin, \(F\) is the noise figure of the amplifier, \(G\) is the gain of the amplifier, and \(P\) is the output power of the amplifier [3, 4]. The second term (noise figure) is constant throughout the spectrum and really only contributes beyond 1 MHz i.e. close in effects are dominated by \(1/f\). As an example if we have a amplifier with 20 dB of gain, a noise figure of 4 dB and output of 1 watt the far out (> 100kHz) contribution would be -147 dBC/Hz. Determining the close in flicker coefficient is not so easy. It depends on the amplifiers transistor characteristics which are not readably available. Fortunately the close in noise effect is small and for our purposes negligible. As an example figure 3 shows the phase noise spectral density of a source (Agilent 4428B) with and without a 20 dB amplifier (Mini-circuits ZFL-2000, NF=7.0). The plots are statistically identical except for the noise figure contribution which can be seen at the bottom of the plot as it reaches 1 MHz. Care must still be taken so that any amplification does not add spurious noise (for example 60 Hz line noise) that can occur with an amplifier through its power supply.
Summary

We have estimated the jitter contributions of two commercial sources and compared them to the CEBAF specification. In addition we have estimated the frequency jitter at the cavity controller and found that amplification effects would be minimal. Even adding the 16 fs of jitter to compensate for the underestimation would put us at approximately 89 fs for the Agilent source. This is over 300 fs below the required jitter of 464 fs. From the results either unit (Agilent or Rhode Schwarz) would be acceptable.

References

2. Walt Kester, “Converting Oscillator Phase Noise to Time Jitter”, Analog Devices MT-008
4. NIST, Physics Laboratory, Time and Frequency Division, http://tf.nist.gov/,